Problem 1. (20 pts.) Assuming that all distances are measured in meters, determine the vector electric field at the coordinate origin given
(a.) a point charge \( Q = 500 \) pC at \((1,1,0)\).
(b.) an infinite length uniform line charge of \( \rho_L = 80 \) pC/m on the line defined by \((x = 0, y = -2)\).
(c.) both the point charge of (a.) and the line charge of (b.).

Problem 2. (30 pts.) A total charge of 1 nC is uniformly distributed over a surface defined by \( 1 \) m \( \leq r \leq 2 \) m, \( 0 \leq \phi \leq 2\pi \) and \( z = 0 \). Determine.
(a.) the surface charge density.
(b.) the absolute potential at the coordinate origin.
(c.) the vector electric field at the coordinate origin.

Problem 3. (25 pts.) The absolute potential in free space is given by
\[
V(x,y,z) = -x^2y(z^2 + 1)
\]
determine
(a.) the vector electric field.
(b.) the vector electric flux density.
(c.) the potential difference \( V_{AB} \) between \( A = (2, 10, 0)m \) and \( B = (2, 1, 0)m \) by integrating the electric field along the straight line path between these two points.

Problem 4. (25 pts.) A spherical charge distribution is defined by
\[
\rho_v = \begin{cases} 
  k & (C/m^3) \quad a \leq R \leq b \\
  0 & (C/m^3) \quad \text{elsewhere}
\end{cases}
\]
where \( k \) is a constant. Using Gauss's law, determine the electric field for
(a.) \( R < a \).
(b.) \( a \leq R \leq b \).
(c.) \( R > b \).
1. (a) \[ E_z = \frac{\sigma}{4\pi \varepsilon_0 |\mathbf{R} - \mathbf{R}'|^3} \] \[ \mathbf{R} - \mathbf{R}' = -\hat{x} - \hat{y} \] \[ E_z = \frac{5 \times 10^{12} (x - y)}{4\pi \varepsilon_0 (8.85 \times 10^{-12})(25)^3} \] \[ \mathcal{E}_z = -1.59 (x + y) \text{ V/m} \]

(b) \[ E_1 = \frac{\partial \phi}{\partial x} \hat{x} = \frac{8 \times 10^{14}}{2\pi \varepsilon_0 (2)} \hat{y} = 0.22 \text{ V/m} \]

(c) \[ E = E_1 + E_2 = (-1.59 x - 0.87 y) \text{ V/m} \]

2. (a) \[ \rho_s = \frac{\rho}{A} = \frac{1}{\pi (a^2 - 1)} = \frac{106.1 \rho C}{m^2} \]

(b) \[ V = \frac{1}{4\pi \varepsilon_0} \int \frac{\rho_s}{R_0} \, ds' = \frac{1}{4\pi \varepsilon_0} \int_0^{2\pi} \int_0^{r} \frac{\rho_s}{R_0} (2\pi)(1) \, dr \, d\phi = \frac{\rho_s}{R_0} \frac{2\pi}{2(8.85 \times 10^{-12})} = 5.99 V \]

(c) \[ \mathcal{E} = \frac{1}{4\pi \varepsilon_0} \int \frac{\rho_s}{R_0^2} \hat{a}_{R_0} \, ds' = 0 \text{ (V/m)} \] by symmetry

3. (a) \[ E = -\nabla V = - \left[ \frac{\partial \mathbf{V}}{\partial x} + \frac{\partial \mathbf{V}}{\partial y} + \frac{\partial \mathbf{V}}{\partial z} \right] = \left[ 2xy \mathbf{z} + 2x^2 \mathbf{z} + 2x^2 \mathbf{z} + 2x^2 \mathbf{z} \right] \text{ V/m} \]

(b) \[ \mathcal{E}_b = \mathcal{E}_0 = \mathcal{E}_0 \left[ 2xy (z^2 + 1) + x^2 (z^2 + 1) + 2x^2 y^2 \right] \text{ V/m} \]

(c) \[ V_{AB} = -\int_{B}^{A} \mathcal{E} \cdot d\mathbf{l} \cdot d\mathbf{l} = (-dy) (-\hat{y}) = dy, \quad \mathcal{E} \cdot d\mathbf{l} = x^2 (z^2 + 1) dy \]

on the given path \((x = 2, z = 0)\), \[ \mathcal{E} \cdot d\mathbf{l} = 4 dy \]

4. \( S_1 \) - spherical surface of radius \( r < a \)

\( S_2 \) - spherical surface of radius \( a < r < b \)

\( S_3 \) - spherical surface of radius \( r > b \)

by symmetry, \( \mathcal{E} \) has only an \( \hat{r} \) component

\( \mathcal{E} \) is uniform on any spherical surface

\[ \mathcal{E} \cdot d\mathbf{s} = \mathcal{E} \cdot \hat{r} \, ds = q \mathcal{E} \cdot d\mathbf{s} = q \mathcal{E} (4\pi r^2) = \mathcal{Q}_{\text{enc}} \Rightarrow \mathcal{E} = \frac{\mathcal{Q}_{\text{enc}}}{4\pi \varepsilon_0 r^2} \]

(a) on \( S_1 \), \( \mathcal{Q}_{\text{enc}} = 0 \), \[ \mathcal{E} = 0 \quad (r < a) \]

(b) on \( S_2 \), \( \mathcal{Q}_{\text{enc}} = \int \int \rho d\mathbf{v} = \int \int [2\pi \int_0^b \sin \theta \, R \, dR \, d\theta] \varepsilon_0 \mathcal{E}_0 \left[ \frac{R^2}{3} \right] \]

\[ Q_{\text{enc}} = \frac{4\pi \varepsilon_0}{3} (R^2 - a^2) \] \[ \mathcal{E} = \left[ \frac{4\pi \varepsilon_0}{3} (R^2 - a^2) / 4\pi \varepsilon_0 R^2 \right] \hat{R} = \left[ \frac{k}{3\varepsilon} \frac{b^3 - a^3}{R^2} \right] \hat{R} \text{ V/m} \]

(c) on \( S_3 \), \( \mathcal{Q}_{\text{enc}} = \int \int \int \varepsilon_0 \mathcal{E} (R^2 - a^2) \sin \theta \, R \, dR \, d\theta \, d\phi = \int \int \int \varepsilon_0 \mathcal{E} \left[ \frac{R^2}{3} \right] \left[ \frac{R^2}{3} \right] \]

\[ Q_{\text{enc}} = \frac{8\pi \varepsilon_0}{3} \] \[ \mathcal{E} = \left[ \frac{4\pi \varepsilon_0}{3} (b^3 - a^2) / 4\pi \varepsilon_0 R^2 \right] \hat{R} = \left[ \frac{k}{3\varepsilon} \frac{b^3 - a^2}{R^2} \right] \hat{R} \text{ V/m} \quad (a < R < b) \]