Problem 1. (20 pts.) A copper ($\sigma_c = 5.8 \times 10^7 \Omega/m$) rectangular waveguide ($a < 2b$) filled with a dielectric characterized by $\mu_r = 1, \varepsilon_r = 2.5$ and $\sigma_d = 0.001 \Omega/m$ is designed to provide single mode operation over the frequency range of 2 GHz to 3 GHz. Determine

(a.) the dimensions ($a, b$) of the waveguide.
(b.) the guide wavelength at $f = 2.5$ GHz.
(c.) the propagation constant at $f = 2.5$ GHz.
(d.) the overall waveguide attenuation in dB/m at $f = 2.5$ GHz.

Problem 2. (15 pts.) An air-filled aluminum ($\sigma_c = 3.5 \times 10^7 \Omega/m$) circular waveguide has a dominant mode cutoff frequency of 8 GHz. Determine

(a.) the radius of the waveguide.
(b.) the waveguide attenuation in dB/m for the dominant TE$_{11}$ mode at $f = 1.2f_c$.
(c.) the waveguide attenuation in dB/m for the low-loss TE$_{01}$ mode at $f = 1.2f_c$.

Problem 3. (15 pts.) An air-filled rectangular cavity has dimensions of $a=3$ cm, $b=2$ cm and $c=5$ cm. Determine

(a.) the lowest resonant frequency of the TM$_{mn0}$ modes and identify that mode.
(b.) the lowest resonant frequency of the TE$_{mnp}$ modes and identify that mode.
(c.) the dominant cavity mode.

Problem 4. (20 pts.) A 6 GHz parallel polarized plane wave in air [region 1, $z < 0$] is incident on a lossless dielectric [region 2, $z > 0$, $\varepsilon_r = 4.2\varepsilon_0, \mu_r = \mu_0$] at an incident angle of 30°. The amplitude of the incident wave electric field is 1 V/m. Determine

(a.) the transmission angle.
(b.) the wavelength in both regions.
(c.) the reflection and transmission coefficients.
(d.) the instantaneous transmitted vector electric field.

Problem 5. (20 pts.) A single layer of lossless nonmagnetic dielectric (thickness = 2.5 cm, $\varepsilon_r = 4$) in air is used as a radome.

(a.) Draw the equivalent transmission line model for the single-layer radome.
(b.) Determine a general expression for the frequencies at which the single-layer radome provides total transmission.
(c.) A second layer of lossless nonmagnetic dielectric ($\varepsilon_r = 3$) is added to the radome on the side of the incoming wave. Draw the equivalent transmission line model for the two-layer radome.
(d.) Determine the thickness of the second layer so that the two-layer system provides total transmission at the lowest frequency found in (b).

Problem 6. (10 pts.) A STIN multimode optical fiber (core refractive index = 1.5, cladding refractive index = 1.493) is 1.2km in length. Determine

(a.) the acceptance angle for the fiber in air.
(b.) the maximum data rate for this optical fiber.
1. (a) \( f_{c_{10}} = \frac{c}{4\pi e_{r} 2a} = 26168 \), \( a = \frac{3 \times 10^5}{2.5 \times 10^9} = 4.74 \text{ cm} \)

\( f_{c_{10}} = \frac{c}{\sqrt{\mu_{e_{r}} 2b} = 3618 \text{ Hz} \), \( b = \frac{2(2.5 \times 10^9)}{3 \times 10^8} = 3.16 \text{ cm} \)

(b) \( \lambda_{10} = \frac{\omega}{f_{10}} = \sqrt{1 - \left(\frac{f_{c_{10}}}{f_{c_{10}}}\right)^2} \), \( \omega = 2\pi f_{c_{10}} = 3.79 \), \( \lambda_{10} = \frac{82.79}{0.07589} = 12.65 \text{ cm} \)

(c) \( \lambda_{0} = \frac{\omega}{f_{0}} = \sqrt{1 - \left(\frac{f_{c_{0}}}{f_{c_{0}}}\right)^2} \), \( \omega = 2\pi f_{c_{0}} = 9.49 \text{ Hz} \), \( \lambda_{0} = \frac{82.79}{0.6} = 9.49 \text{ Hz} \)

(d) \( \lambda_{10} = \frac{\omega}{f_{10}} = \sqrt{1 - \left(\frac{f_{c_{10}}}{f_{c_{10}}}\right)^2} \), \( \omega = 2\pi f_{c_{10}} = 5.53 \times 10^3 \), \( \lambda_{10} = \frac{82.79}{0.013152} = 0.1986 \)

\( \lambda_{0} = \frac{\omega}{f_{0}} = \sqrt{1 - \left(\frac{f_{c_{0}}}{f_{c_{0}}}\right)^2} \), \( \omega = 2\pi f_{c_{0}} = 0.2039 \), \( \lambda_{0} = \frac{82.79}{0.04975} = 1.2 \text{ cm} \)

2. (a) \( f_{c_{11}} = \frac{2\pi a k e}{c} = 8610 \text{ Hz} \), \( a = \frac{8\pi}{2\pi (8 \times 10^3)} = 1.099 \text{ cm} \)

(b) \( \lambda_{11} = \frac{\omega}{f_{11}} = \sqrt{1 - \left(\frac{f_{c_{11}}}{f_{c_{11}}}\right)^2} \), \( \omega = 2\pi f_{c_{11}} = 0.0329 \), \( \lambda_{11} = \frac{82.79}{0.0159} = 0.139 \text{ cm} \)

3. (a) \( f_{r} = \frac{c}{2 \sqrt{(m_1^2 + n_2^2 + (\frac{1}{m} \frac{1}{n})^2)}} \text{ Hz} \)

(b) \( f_{r} = \frac{c}{2 \sqrt{(1.0.03)^2 + (1.0.02)^2}} = 9.01 \text{ Hz} \)

4. (a) \( \Theta_{x} = \sin^{-1}\left(\frac{1}{n} \sin \theta_{x}\right) \), \( \Theta_{y} = \sin^{-1}\left(\frac{1}{n} \sin \theta_{y}\right) = 14.12^\circ \)

(b) \( \lambda_{x} = \frac{c}{2} = \frac{3 \times 10^8}{6 \times 10^9} = 5 \text{ cm} \), \( \lambda_{y} = \frac{c}{2} = 2.44 \text{ cm} \)

(c) \( T = \frac{\pi}{2} \), \( T_{11} = \frac{\pi}{2} \cos \theta_{x} + \frac{n}{\pi} \cos \theta_{y} \)

(d) \( E_{x}^{r} = E_{0} \cos \Theta_{x} \sin \theta_{x} \), \( E_{y}^{r} = E_{0} \sin \theta_{x} \cos \theta_{x} \), \( \lambda_{x} = \frac{2\pi}{2} = 257.5 \)

\( E_{x}^{r} = 0.631 (0.9698 - 0.2440) e^{-\frac{2}{2} (0.2440 + 0.9698)} \)

5. (a) \( \lambda_{0} \)

(b) \( \lambda_{0} = n_0 \frac{\lambda_{0}}{2} \text{(MATCHED)} \)

(c) \( \lambda_{0} = 10^{9} \lambda_{0} \)

(d) \( \lambda_{0} = n_0 \frac{\lambda_{0}}{2} \text{(MATCHED)} \)

6. (a) \( \Theta = \sin^{-1}\left(1.25 \right) \), \( \Theta = \sqrt{1.5^2 - 1.493^2} = 8.32^\circ \)

(b) \( f_{max} = \frac{c}{2 \ln (n_1 - n_2)} = 17.8 \text{ MHz} \)