Problem 1. (20 pts.) An air-filled rectangular waveguide is to be designed such that the cutoff frequencies of the TM_{11} and TE_{20} modes are both equal to 12 GHz. Determine
(a.) the waveguide dimensions \([a, b]\).
(b.) the frequency range for single mode operation \([f_{\text{min}}, f_{\text{max}}]\).
(c.) the group velocity when the waveguide is operated at the frequency midway between \(f_{\text{min}}\) and \(f_{\text{max}}\).
(d.) the waveguide wavelength at the operating frequency of part (c.).

Problem 2. (10 pts.) A copper \((\sigma_c = 5.8 \times 10^7 \ \Omega/m)\) circular waveguide of radius 0.9 cm is filled with a lossless nonmagnetic dielectric \((\varepsilon_r = 4)\). Determine
(a.) the cutoff frequency of the dominant mode.
(b.) the waveguide attenuation in dB/m due to conductor losses for the TE_{01} mode when the waveguide is operated at \(f = 1.2f_{c01}\).

Problem 3. (10 pts.) A rectangular cavity \((a = 4.8 \ \text{cm}, b = 2.6 \ \text{cm}, c = 3.9 \ \text{cm})\) is air-filled.
(a.) Determine the lowest resonant frequency and identify the mode.
(b.) If this cavity is filled with a lossless dielectric of relative permittivity \(\varepsilon_r\), find the value of \(\varepsilon_r\) that would shift the resonant frequency to 70% of the air-filled value.

Problem 4. (25 pts.) Perpendicularly polarized light (wavelength = 500 nm) in air [region 1, \(z < 0\)] is incident on glass [region 2, \(z > 0, n_2 = 1.5\)] at an incident angle of 20°. The amplitude of the incident wave electric field is 6 V/m. Determine
(a.) the wavelength in the glass.
(b.) the transmission angle.
(c.) the reflection and transmission coefficients.
(d.) the transmitted vector electric field phasor.
(e.) the instantaneous transmitted vector electric field.

Problem 5. (25 pts.) A single layer of lossless nonmagnetic dielectric \((\text{thickness} = 1.2 \ \text{cm})\) is used as a radome in air. The permittivity \(\varepsilon\) of the dielectric is to be determined.
(a.) Determine the minimum permittivity \([\varepsilon_{\text{min}}]\) that provides radome operation at 8 GHz.
(b.) What is the radome thickness in wavelengths of the radome with \(\varepsilon = \varepsilon_{\text{min}}\) at 8 GHz?
(c.) Determine all frequencies at which the single layer \((\varepsilon = \varepsilon_{\text{min}})\) provides radome operation.
(d.) A second layer of lossless nonmagnetic dielectric is added to the radome on the side of the incoming wave. The permittivity of the second layer is 1.4\(\varepsilon_{\text{min}}\). Determine the minimum thickness of the second layer for radome operation at 8 GHz.

Problem 6. (10 pts.) The minimum free-space wavelength for single mode operation in a given STIN optical fiber \((\text{core diameter} = 9 \ \mu\text{m}, \text{core refractive index} = 1.51)\) is 1250 nm. Determine
(a.) the maximum allowable numerical aperture.
(b.) the minimum allowable value of the cladding refractive index.
1. (a) \( f_{cm} = \frac{c}{2\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{b}{c}\right)^2}} \), \( f_{ce} = \frac{f_{cm}}{2a} \), \( a = \frac{c}{f_{ce}} = \frac{3\times10^8}{12\times10^9} = 2.5 \text{ cm} \)

\( f_{cm} = \frac{c}{2\sqrt{\frac{1}{a^2} + \frac{2}{b^2}}} \), \( b = \frac{2f_{cm}}{c} \), \( b = \frac{1}{\sqrt{820^2 - (10.025)^2}} = 1.44 \text{ cm} \)

(b) \( f_{ce, 15} = \frac{c}{2a} = \frac{3\times10^8}{2(0.025)} = 6 \text{ GHz} \), \( \theta_{01} = \frac{c}{2b} = 10.42 \text{ GHz} \)

(c) \( f = \frac{6\times10.42}{2} = 8.216 \text{ GHz} \), \( u_{0} = \frac{c}{\sqrt{1 - \left(\frac{f}{f_{cm}}\right)^2}} = \frac{3\times10^8}{\sqrt{1 - \left(\frac{6}{2.5}\right)^2}} = 5.35 \text{ cm} \)

\( \lambda_0 = \frac{\lambda}{\sqrt{1 - \left(\frac{f_{cm}}{f}\right)^2}} \), \( \lambda' = \frac{\lambda}{\sqrt{1 - \left(\frac{f_{cm}}{f_{ce, 15}}\right)^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{2.5}{6}\right)^2}} = 4.88 \text{ GHz} \)

2. (a) \( f_{cm} = \frac{2\pi c M}{\lambda_{0}} \), \( f_{ce} = \frac{2\pi c M}{\lambda'} \), \( \lambda_{0} = \frac{2\pi c (0.009)^{1/2}}{1.8412 \times 3.8210^{10}} = 10.16 \text{ GHz} \), \( f = 1.2f_{cm} \)

(b) \( \alpha_{cm, 15} = \frac{R_{cm}}{\lambda_{0}} \), \( \alpha_{ce} = \frac{R_{ce}}{\lambda'} \), \( \lambda_{0} = \frac{2\pi c (0.009)^{1/2}}{1.8412 \times 3.8210^{10}} = 10.16 \text{ GHz} \), \( f = 1.2f_{cm} \)

\( \mu_s = 0.598 \mu_m \), \( R_x = \frac{c}{\lambda_{0}} = 0.028 \Omega \), \( h_n = \frac{n_0}{q} \), \( \lambda_0 = 0.019 q \), \( n_0 = -0.1853 \mu_m \)

3. (a) \( a > c > b \), \( f_{101} = \frac{c}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 + \left(\frac{c}{2}\right)^2}} = 4.96 \text{ GHz} \), \( \mu_{101} \)

(b) \( f_{101} \) (FILLED) = 0.7 \( f_{101} \) (FILL) \( = \frac{1}{\sqrt{c_{101}}} \), \( f_{101} \) (FILL) \( = \frac{1}{\sqrt{c_{101}}} \), \( \mu_{101} \) \( = \frac{1}{\sqrt{1.5}} \), \( \mu_{101} = 3.33 \mu_m \)

4. (a) \( \frac{\lambda_0}{\lambda_0} = \frac{u_0}{f} = \frac{v}{n_0} \), \( \lambda_0 = \frac{n_0}{\lambda_2} \), \( \lambda_2 = \frac{n_0}{\lambda_2} \)

(b) \( \theta = \sin^{-1} \left( \frac{n_2}{n_2} \sin \theta_{02} \right) = \sin^{-1} \left( \frac{1}{1.5} \sin 20^\circ \right) = 13.1^\circ \)

(c) \( T_1 = \frac{n_2 \cos \theta_{01} - n_1 \cos \theta_{02}}{n_2 \cos \theta_{01} + n_1 \cos \theta_{02}} \), \( \theta_1 = \frac{n_0}{1.5} \), \( \theta_1 = -0.21^\circ \)

\( T_1 = \frac{n_2 \cos \theta_{01} - n_1 \cos \theta_{02}}{n_2 \cos \theta_{01} + n_1 \cos \theta_{02}} = 0.783 \)

\( k_x = \frac{2\pi}{a_x} = 1.825 \times 10^7 / \mu_m \)

(d) \( \mu_{101} \) = \( \frac{\mu_{101}}{\mu_{101}} \) \( = 4.7 \mu \) \( \frac{2}{\mu_{101}} \) \( \frac{1}{\mu_{101}} \) \( \mu_{101} \) \( \mu_{101} \) \( \mu_{101} \) \( \mu_{101} \)

(e) \( \mu_{101} \) \( = \frac{R_{1} \mu_{101}}{\mu_{101}} \) \( \mu_{101} \) \( \mu_{101} \) \( \mu_{101} \) \( \mu_{101} \)

5. (a) \( d_1 = 1.2 \text{ cm} \) \( = n d_2 = n \frac{c}{2 f_{cm}} \), \( \theta_1 = \frac{2\pi}{2 f_{cm}} \)

\( \theta_1 = \frac{2\pi}{2 f_{cm}} = \frac{3\times10^8}{12\times10^9} = 2.44^\circ \)

6. (a) \( N_A < \frac{2.405 \times 10^7}{2 \pi \mu_m} = \frac{2.405 \times 1250 \times 10^9}{2 \pi (4.5 \times 10^8)} = 0.1063 \), \( N_A < 0.1063 \)

(b) \( N_A > \frac{\sqrt{n_1^2 - n_2^2} - 0.1063}{n_1^2 - n_2^2}, n_2 > n_2^2 + 0.1063^2 \)

\( n_2 > \sqrt{n_1^2 - (0.1063)^2} = \sqrt{1.51^2 - (0.1063)^2} = 1.5063 \)

\( n_2 > 1.5063 \)