Problem 1. (25 pts.) The region between the conductors of a spherical capacitor (inner radius $a = 5$ cm, outer radius $b = 10$ cm) is characterized by $\varepsilon = 2\varepsilon_0$ and $\rho_s = -4\varepsilon_0 R \, (C/m^3)$. The inner conductor is held at a potential of $+2$ V while the outer conductor is held at $-2$ V. Assuming the potential varies only in the radial direction, determine
(a.) the potential as a function of $R$ between the conductors ($a < R < b$).
(b.) the vector electric field as a function of $R$ between the conductors ($a < R < b$).
(c.) the capacitance of the spherical capacitor.

Problem 2. (25 pts.) A square grid with $\Delta x = \Delta y = 1$ cm is shown for the 2-D conductor system in air shown below. A finite-difference solution is sought for the potential in the charge-free region between the conductors.
(a.) Write the finite-difference equations necessary to solve for the potential at all free nodes $V_i^k$ where $i$ is the grid point index and $k$ is the iteration number.
(b.) Determine the free node voltages for the first two iterations assuming an initial guess of zero for all free node voltages.

Problem 3. (25 pts) A product is tested for FCC Class B conducted emissions compliance at 1 MHz and radiated emissions compliance at 500 MHz. The DUT generates conducted noise currents of 14.5 dB$\mu$A (average) at 1 MHz. The 500 MHz radiated emissions measurement is performed at a distance of 13 ft between the DUT and the measurement antenna. The antenna factor of the measurement antenna is 15.3 dB and is connected to the spectrum analyzer by 20 ft of coaxial cable (attenuation = 12.9 dB/100 ft at 500 MHz). The voltage measured by the spectrum analyzer is 27.8 dB$\mu$V. Determine
(a.) the magnitude of the power (dBm) at the spectrum analyzer input.
(b.) the magnitude of the voltage (dB$\mu$V) at the measurement antenna terminals.
(c.) the electric field magnitude (dB$\mu$V/m) at the measurement antenna.
(d.) if the product passes or fails radiated emissions compliance and by how much.
(e.) if the product passes or fails conducted emissions compliance and by how much.

Problem 4. (25 pts.) A pair of parallel wires (length = 20 cm, separation = 4 mm) carry 300 MHz currents of 68.9 and 70.2 mA in opposite directions. Determine
(a.) the magnitudes (dB$\mu$A) of the differential-mode and common-mode currents.
(b.) the radiated electric field magnitude (dB$\mu$V/m) due to the common-mode currents at a distance of 8 m in the direction of maximum radiation.
(c.) the radiated electric field magnitude (dB$\mu$V/m) due to the differential-mode currents at a distance of 8 m in the direction of maximum radiation.
1. (a) \[ \frac{V^2}{E} = \frac{1}{R^2} \frac{d}{dR} \left( \frac{R^2dV}{dR} \right) = \frac{R}{2} \frac{d}{dR} \left( \frac{R^2dV}{dR} \right) = 2R, \]
\[ R \frac{d^2V}{dR^2} = R^2 + C_1, \quad \frac{dV}{dR} = 1 + \frac{C_1}{R^2}, \quad V(R) = -\frac{C_1}{R} + C_2 \]
\[ V(0.05) = 0.05 - 0.05C_1 + C_2 = 0.75 \quad \therefore C_1 = -0.405 \]
\[ V(0.10) = 0.10 - 0.10C_1 + C_2 = -2 \quad \therefore C_2 = -6.15 \]
\[ V(R) = R \left[ \frac{0.405}{R^2 - 1} \right] \frac{1}{R} \]

(b) \[ E(R) = -R^2 \frac{d}{dR} \frac{1}{R^2} (1 + \frac{C_1}{R^2}) = \left( \frac{0.405}{R^2 - 1} \right) \frac{V}{m} \]

(c) \[ C = \frac{Q}{V}, \quad V = 4V, \quad Q = (p + A_+), \quad p_+ = 0 \quad \text{at} \quad (0, 0.05) = 260 \quad \text{at} \quad (0.05, 0.05) = 2.85 \quad \text{at} \quad (0.05, 0.05) = 0.0314 \]
\[ Q = (2.85 \times 10^{-9}) (0.0314) = 89.5 \quad \therefore C = \frac{89.5}{V} \quad \text{at} \quad (0.05, 0.05) = 22.4 \mu F \]

2. (a) \[ \frac{d^2V}{dR^2} = 0 \quad \Rightarrow \quad V_{(R)} = \frac{1}{4} \left[ V_{(R)}^{(k)} + V_{(R)}^{(i)} + V_{(R)}^{(k)} + V_{(R)}^{(i)} \right] \]
\[ V_{(R)}^{(k)} = \frac{1}{4} \left[ 30 + V_{(R)}^{(k)} \right] \quad (b) \quad V_{(R)}^{(i)} = \frac{30}{4} = 7.5 \quad (c) \quad V_{(R)}^{(i)} = \frac{32.5}{4} = 8.125 \]
\[ V_{(R)}^{(k)} = \frac{1}{4} \left[ 10 + V_{(R)}^{(k)} + V_{(R)}^{(k)} \right] \quad V_{(R)}^{(i)} = \frac{10}{4} = 2.5 \quad V_{(R)}^{(k)} = \frac{10}{4} = 2.5 \]
\[ V_{(R)}^{(k)} = \frac{1}{4} \left[ 10 + V_{(R)}^{(k)} + V_{(R)}^{(k)} \right] \quad V_{(R)}^{(i)} = \frac{10}{4} = 2.5 \quad V_{(R)}^{(k)} = \frac{10}{4} = 2.5 \]
\[ V_{(R)}^{(k)} = \frac{1}{4} \left[ -10 + V_{(R)}^{(k)} + V_{(R)}^{(k)} \right] \quad V_{(R)}^{(i)} = \frac{-10}{4} = -2.5 \quad V_{(R)}^{(i)} = \frac{-10}{4} = -2.5 \]
\[ V_{(R)}^{(k)} = \frac{1}{4} \left[ -30 + V_{(R)}^{(k)} \right] \quad V_{(R)}^{(i)} = \frac{-32.5}{4} = -8.125 \]

3. (a) \[ R_m = \frac{V_m^2}{R} = \left[ \frac{10^6 \times 10^{19} \times 10^{19}}{20} \right] = 12.05 \Omega \quad \text{pW} = -79.2 \text{ dBm} \]

(b) \[ \text{Cable Gain} = \frac{(-12.9 \text{ dB} / 100 \text{ ft}) \times (20 \text{ ft})}{100} = -2.58 \text{ dB} \]
\[ V_{ant,db} + \text{cable gain}_{db} = V_{ant,db} = 29.8 + 2.58 = 30.4 \text{ dB} \mu V \]

(c) \[ E_{ant, db} + \text{cable gain}_{db} = E_{ant, db} + 20 \log_{10} \left( \frac{3.96}{3} \right) = 48.7 + 2.41 = 48.1 \text{ dB} \mu V / \text{m} \]

(d) \[ E_R(=3) = E(=3.96) \times 2 \log_{10} \left( \frac{3.96}{3} \right) = 48.1 + 2.41 = 48.1 \text{ dB} \mu V / \text{m} \]

4. (a) \[ I_0 = \frac{1}{2} \left( 68.9 \times 10^{-2} \right) = 69.55 \text{ mA} = 96 \text{ dB} \mu A \]
\[ I_c = \frac{1}{2} \left( 70.2 \times 10^{-2} \right) = 69.55 \text{ mA} = 56.3 \text{ dB} \mu A \]

(b) \[ I_{(m)} = 1.25 \left( \frac{1}{2} \right) I_c = 1.25 \left( 3 \times 10^{15} \right) \frac{0.2}{8} (0.65 \times 10^{-3}) = 6127 \mu \text{A} \]

(e) \[ I_{(m)} = 1.316 \times 10^{-3} \times \frac{1}{2} I_0 = 1.316 \times 10^{-2} (3 \times 10^8)^2 (0.2)(0.004)(69.55 \times 10^{-3}) \]
\[ = 8238 \mu \text{V} / \text{m} = 78.3 \text{ dB} \mu \text{V} / \text{m} \]