INDUCTION (ASYNCHRONOUS) MACHINES

The three-phase induction machine is the most widely-used rotating machine in industry. Induction machines are almost always operated as motors due to their undesirable characteristics as a generator. Single-phase induction motors are also very commonplace, being used for most household applications requiring a motor.

The induction motor gets its name from the manner in which it operates. An external AC current is provided to the stator windings of an induction motor but no external current is provided to the rotor windings. The AC currents that result in the rotor of an induction motor are the result of induction (Faraday’s law). An induction motor is also classified as an asynchronous motor due to the fact that its operating speed is slightly less than synchronous speed. A machine is classified as a synchronous machine if its operating speed is directly proportional to the frequency of the electrical system. The speed of operation for a synchronous machine is thus designated as synchronous speed.

The induction motor has many advantages:

1. The induction motor is rugged, inexpensive and easy to maintain.
2. Induction motors range in size from a few watts to 10,000hp.
3. The speed of an induction motor is nearly constant. The speed typically varies only by a few percent going from no load to rated load.

Some disadvantages of the induction motor are:

1. The speed is not easily controlled.
2. The starting current may be five to eight times the full-load current.
3. The lagging power factor is low when the machine is lightly loaded.
THREE-PHASE INDUCTION MOTOR OPERATION

In a three-phase induction motor, three-phase voltages are applied to the stator windings. This results in balanced three-phase currents flowing in the stator windings. Based on the geometry of the stator coils and the resulting currents, a rotating magnetomotive force (mmf) is produced by the stator in the rotor (see Figure 6.6, p. 312). The rotation speed of the stator mmf is dependent on the number of poles \( p \) in the stator winding. The number of poles is always an even number since the magnetic poles always occurs in pairs. The stator mmf rotates at a rate of \( 2/p \) revolutions per period (period of the stator current \( = T_1 = 1/f_1 \)). The speed of the stator mmf rotation is defined as the synchronous speed \( n_s \). In terms of revolutions per minute (rpm), the speed of the stator mmf rotation, and thus, the synchronous speed is

\[
  n_s = 120 \frac{f_1}{p} \text{ (rpm)}
\]

where \( f_1 \) is the frequency of the stator current.

The rotating stator mmf is applied to the rotor through the air gap between the stator and the rotor. There are two types of rotors used in induction motors.

Rotor Types

1. **Squirrel-cage rotor** - conducting bars that are electrically connected at both ends of the rotor by end rings.
2. **Wound rotor** - polyphase windings connected to slip rings at both ends of the rotor.

Both rotor types are contained in slots in a laminated core which is mounted on the motor shaft.
The conductors of the rotor experience the rotating mmf of the stator. The orientation of the rotor conductors relative to the rotating stator mmf produce an electromotive force along the conductor according to

\[ V_{emf} = \int_L (u \times B_s) \cdot dl \]  

(motional induction)  

"flux cutting emf"

where \( u \) is the vector velocity of the rotor conductors relative to the stator magnetic flux density \( (B_s) \). This electromotive force (emf) induces currents in the rotor conductors \( (I_r) \). These current carrying rotor conductors in the applied stator magnetic flux experience a vector force given by

\[ F = \int_L (I_r \times B_s) \cdot dl \]

These forces on the conductors of the rotor set the rotor in motion. There is an upper limit on the speed of the induction motor. If the rotor were turning at the synchronous speed (the same speed as the stator mmf), there would be no relative velocity between the rotor conductors and the stator mmf. This would result in zero emf along the rotor conductors, no current in the rotor conductors, and no force on the rotor conductors. Thus, the induction motor never reaches synchronous speed and operates at some speed less than synchronous speed.

The difference between the motor speed \( (n) \) and the synchronous speed \( (n_s) \) is defined as the slip speed \( (n_{slip}) \) and given by

\[ n_{slip} = n_s - n \text{ (rpm)} \]

If the slip speed is defined on a per-unit basis (normalized to the synchronous speed), the resulting value is defined as the slip \( (s) \).

\[ s = \frac{n_s - n}{n_s} \]

Note that if the motor speed equals the synchronous speed, \( s = 0 \). If the motor is stationary, \( s = 1 \). The slip speed in rpm can be written in terms of
the slip as \( n_{\text{slip}} = s n_s \).

The frequency of the current and voltage in the rotor circuit \( f_2 \) is dependent on the relative speed between the stator mmf and the machine speed \( (n_s - n) \). Using the relationship between frequency and rotation speed in rpm, we may write

\[
n_2 = n_s - n = 120 \frac{f_2}{p} \quad \text{(rpm)}
\]

where \( n_2 \) represents the rotation speed of the field produced by the induced currents in the windings relative to the rotor speed. Solving for the frequency of the rotor signals gives

\[
f_2 = \frac{p}{120} (n_s - n)
\]

\[
= \frac{p}{120} s n_s
\]

\[
= sf_1
\]

Thus, the frequency of the signals in the rotor equals the frequency of the excitation in the stator times the slip. This frequency is commonly referred to as the slip frequency.

Given that the rotor is rotating at the machine speed of \( n \) rpm, the induced rotor field rotates in the air gap at a speed of \( n + n_2 = n_s \). Therefore, both the stator field and the rotor field rotate in the air gap at the synchronous speed.
Example (Induction machine operation)

A three-phase, 460 V, 100 hp, 60 Hz four-pole induction machine delivers rated output power at a slip of 0.05. Determine the
(a.) synchronous speed and motor speed.
(b.) speed of the rotating air gap field.
(c.) frequency of the rotor circuit.
(d.) slip rpm.
(e.) speed of the rotor field relative to the rotor structure, the stator structure, and the stator rotating field.

(a.) \[ n_s = 120 \frac{f_1}{p} = 120 \frac{60}{4} = 1800 \text{ rpm} \]

\[ n = n_s (1 - s) = 1800 (1 - 0.05) = 1710 \text{ rpm} \]

(b.) 1800 rpm \(n_s\)

(c.) \[ f_2 = s f_1 = (0.05)(60) = 3 \text{ Hz} \]

(d.) \[ n_{\text{slip}} = s n_s = (0.05)(1800) = 90 \text{ rpm} \]

(e.) relative to the rotor structure = 90 rpm
relative to the stator structure = 1800 rpm
relative to the stator rotating field = 0 rpm
**STATOR AND ROTOR INDUCED VOLTAGES**

The relationship between the induced voltage (rms) in a winding around a ferromagnetic core carrying a sinusoidal magnetic flux density was found to be

\[ V = 4.44 N f B_{\text{max}} A \]

where \( N \) is the number of turns in the winding, \( f \) is the frequency of the sinusoidal flux density, \( B_{\text{max}} \) is the peak value of the magnetic flux density in the core and \( A \) is the cross-sectional area of the core. This equation can also be applied to the stator and rotor of the three-phase induction machine. First, the equation above was based on the assumption that the flux density in the core was uniform. This will not be the case in the induction machine (rotating mmf). We can replace the \( B_{\text{max}} A \) term (total magnetic flux for a uniform flux density) by the more general total magnetic flux term \( \psi_m \).

\[ V = 4.44 N f \psi_m \]

The total magnetic flux in the air gap between the stator and the rotor will depend on the number of poles \( p \) for the induction machine. However, windings associated with each pole will be connected in parallel and produce the same induced voltage. Thus, the total magnetic flux needed in the induced voltage equation is the total magnetic flux per pole \( \psi_{mp} \). The number of turns in each case will be \( N_1 \) (stator winding) and \( N_2 \) (rotor winding) and represent the number of turns per phase. Also, the equation for the stator and the rotor should include the frequency of the magnetic flux density through the respective winding. Based on these factors, the equations for the induced voltage in the stator and the rotor of the induction machine are

\[ V_1 = 4.44 N_1 f_1 \psi_{mp} \]
\[ V_2 = 4.44 N_2 f_2 \psi_{mp} \]
There are additional geometric factors regarding how the windings are arranged that affect (reduce) the induced voltages. These reduction factors are included in the form of the *winding factor* $K_w$. The induced voltages for the stator and the rotor are then

$$V_1 = 4.44N_1 f_1 \psi_{mp} K_{w1}$$

$$V_2 = 4.44N_2 f_2 \psi_{mp} K_{w2}$$

where $K_{w1}$ and $K_{w2}$ are the winding factors for the stator and the rotor, respectively.

If the rotor of three-phase induction machine is open-circuited so that no induced current can flow in the rotor, and a three-phase source is applied to the stator, the stator field rotates at the synchronous speed while the rotor stays stationary (standstill). This corresponds to a slip of $s = 1$ and an induced voltage in the rotor at a frequency of

$$f_2 = sf_1 = f_1$$

is

$$V_{2,\text{standstill}} = 4.44N_2 f_1 \psi_{mp} K_{w2}$$

With the induced voltages in both the stator and the rotor at the same frequency, the ratio of the stator voltage to the rotor voltage reduces to

$$\frac{V_{1,\text{standstill}}}{V_{2,\text{standstill}}} = \frac{4.44N_1 f_1 \psi_{mp} K_{w1}}{4.44N_2 f_1 \psi_{mp} K_{w2}} = \frac{N_1 K_{w1}}{N_2 K_{w2}}$$

The winding factors for the stator and the rotor for most induction machines are normally equal, so that

$$\frac{V_{1,\text{standstill}}}{V_{2,\text{standstill}}} \approx \frac{N_1}{N_2}$$

Thus, the turns ratio of the induction machine is equivalent to the ratio of the stator induced voltage to the rotor induced voltage at standstill.
When the induction machine is operating at a slip $s$, the frequency in the rotor circuit is the slip frequency:

$$ f_2 = s f_1 $$

The induced voltage in the rotor at the slip frequency is

$$ V_{2, \text{slip}} = 4.44 N_2 f_2 \psi_{mp} K_{w2} $$
$$ = 4.44 N_2 s f_1 \psi_{mp} K_{w2} $$
$$ = s V_{2, \text{standstill}} $$
$$ = s \frac{N_2}{N_1} V_{1, \text{standstill}} = s \frac{N_2}{N_1} V_1 $$

Thus, the induced voltage in the rotor when the machine operates at a slip of $s$ is equal to the slip times the induced rotor voltage at standstill.

Example

A three-phase, 460 V, 100 hp, 60 Hz four-pole induction machine delivers rated output power at a slip of 0.05. The stator windings are wye-connected and the machine turns ratio is 1:0.5. Determine the induced voltage in the rotor windings.

$$ V_1 = \frac{460}{\sqrt{3}} \quad \text{(wye-connected stator)} $$

$$ V_{2, \text{slip}} = s \frac{N_2}{N_1} V_1 $$
$$ = (0.05)(0.5) \frac{460}{\sqrt{3}} $$
$$ = 6.64 \text{ V} $$
INDUCTION MACHINE MODES OF OPERATION

There are three modes of operation for the induction machine. These modes are defined by specific ranges of slip and speed.

1. Motor \(0 \leq s \leq 1\) \(0 \leq n \leq n_s\)
2. Generator \(s < 0\) \(n > n_s\)
3. Brake \(s > 1\) \(n < 0\)

Motor

The natural mode of operation for the induction machine is as a motor. The motor speed is between zero and the synchronous speed while the slip is between 0 and 1.

Generator

The induction machine may be operated as a generator if the stator is connected to a constant frequency voltage source and the rotor is then driven above synchronous speed in the same direction as the stator rotating mmf by some external source. The slip of the induction machine is negative when operated as a generator.

Brake

Given an induction motor running at normal speed, if the leads on the stator windings are reversed suddenly, the direction of rotation for the stator field is reversed. The resulting slip is larger than one. The motor will come to an abrupt stop since the force on the rotor now opposes the normal rotor rotation. The motor must then be disconnected from the voltage source before it starts to rotate in the reverse direction. This method of bringing motors to a quick stop is commonly known as plugging.
**THREE-PHASE INDUCTION MACHINE EQUIVALENT CIRCUIT**

From the standpoint of the operation, the stator winding of the induction machine performs basically the same function of the primary winding in a transformer. A current is driven through the stator windings which produces a magnetic flux in the magnetic core (stator/air gap/rotor). The stator will have a winding resistance and a leakage reactance along with a core resistance and a magnetization reactance. The per-phase equivalent circuit for the stator winding is shown below. Note that the stator winding reactances are defined in terms of the stator source frequency $\omega_1 = 2\pi f_1$.

![Stator Winding Equivalent Circuit](image)

- $V_1$ - per-phase stator terminal voltage
- $R_{w1}$ - per-phase stator winding resistance
- $X_{l1} = \omega_1 L_{l1}$ - per-phase stator leakage reactance
- $R_{c1}$ - per-phase stator core loss resistance
- $X_{m1} = \omega_1 L_{m1}$ - per-phase stator magnetizing reactance
- $E_1$ - per-phase induced voltage in the stator winding
One significant difference between the transformer equivalent circuit and the induction machine equivalent circuit is the magnitude of the current in the excitation branch. In transformers, this current was small in comparison to primary current. In induction machines, this current is significant (up to 50% of the rated input current). The leakage reactance of an induction machine is also larger than that of a transformer primarily due to the air gap.

The equivalent circuit of the rotor consist of the rotor winding resistance in series with the rotor leakage reactance. The rotor equivalent circuit at the slip frequency is shown below.

\[ jX_{l2} = j\omega_2 L_{l2} = Js\omega_1 L_{l2} \]

\[ I_2 \]

\[ V_{2, \text{slip}} = sV_{2, \text{standstill}} \]

\[ R_{w2} \]

Rotor Winding Per-phase Equivalent Circuit (\(\omega=\omega_2\))

According to the equivalent rotor circuit, the rotor current is

\[ I_2 = \frac{sV_{2, \text{standstill}}}{R_{w2} + js\omega_1 L_{l2}} = \frac{V_{2, \text{standstill}}}{\frac{R_{w2}}{s} + j\omega_1 L_{l2}} \]

This expression for the rotor current is equivalent to the following equivalent circuit.
Note that this equivalent circuit represents the rotor as seen from the stator since the circuit is now at the stator frequency \( \omega_1 \). The power dissipated in the equivalent circuit resistance \( R_{w2}/s \) includes both copper losses in the rotor and the developed mechanical power. The equivalent circuit resistance can be represented as the sum of the rotor winding resistance and the developed mechanical power resistance as follows.

\[
\frac{R_{w2}}{s} = \frac{R_{w2}}{s} + (R_{w2} - R_{w2})
\]

\[
= R_{w2} + \left( \frac{R_{w2}}{s} - R_{w2} \right)
\]

\[
= R_{w2} + \frac{R_{w2}}{s} (1 - s)
\]

The final form of the rotor per-phase equivalent circuit is shown below.
Using this form of the rotor winding per-phase equivalent circuit, we may identify the rotor copper loss and the total real mechanical power developed by the induction machine ($P_{\text{mech}}$).

\[ P_{\text{Cu2}} = I_2^2 R_{w2} \]

\[ P_{\text{mech}} = I_2^2 R_{w2} \frac{1 - s}{s} = P_{\text{rot}} + P_{\text{out}} \]

The equation for the mechanical power shows that the induction motor should be operated at a small value of slip for efficient operation. The power available as output shaft power ($P_{\text{out}}$) for the induction motor is found by subtracting the rotational losses ($P_{\text{rot}}$ - windage and friction) from the developed mechanical power.

The overall per-phase equivalent circuit for the induction machine can be found by reflecting the rotor circuit back to the stator side by using the turns ratio for the machine. The resulting equivalent circuit is shown below.

![Complete Induction Machine Per-Phase Equivalent Circuit](image)

**Complete Induction Machine Per-Phase Equivalent Circuit**

\[ X_{l2}' = a^2 X_{l2} \]
\[ R_{w2}' = a^2 R_{w2} \]
\[ a = \frac{N_1}{N_2} \]
\[ I_2' = \frac{I_2}{a} \]
APPROXIMATE EQUIVALENT CIRCUITS
(THREE-PHASE INDUCTION MACHINE)

As with the transformer equivalent circuit, the induction machine equivalent circuit can be simplified given certain conditions. If the voltage drop across the stator winding resistance and leakage reactance is small so that the stator input voltage and the stator induced voltage are nearly equal, the excitation branch of the equivalent circuit can be moved to the input terminals as was done for the transformer equivalent circuit.

The individual losses in an induction machine (core loss, friction, windage) will vary with the machine speed. However, the overall loss of these three components remains relatively constant at any speed. Thus, the core loss given by $R_{c1}$ can be removed and lumped with the rotational loss (friction, windage) that is included in the mechanical power term which yields the approximate equivalent circuit below.
For many induction machine problems, the excitation branch current is significant enough that the movement of the excitation branch to the input terminal of the equivalent circuit is not valid. However, the core loss resistance can be lumped with the rotational losses like before. These assumptions lead to the approximate equivalent circuit shown below, which is accurate for most applications.

IEEE-Recommended Induction Machine Per-Phase Approximate Equivalent Circuit