Three-Phase Transformers

Transformers used in three-phase systems may consist of a bank of three single-phase transformers or a single three-phase transformer which is wound on a common magnetic core. A three-phase transformer wound on a common core offers advantages over a bank of single-phase transformers. A three-phase transformer wound on a common core is lighter, smaller and cheaper than the bank of three single-phase transformers. The common core three-phase transformer also requires much less external wiring than the bank of single-phase transformers and can typically achieve a higher efficiency.

The bank of three single-phase transformers does offer the advantage of flexibility. In the case of an unbalanced load, one or more transformer in the bank can be replaced by a larger or smaller kVA-rated transformer. In terms of maintenance, a malfunctioning transformer in the bank of transformers can be easily replaced while the entire common core three-phase transformer would require replacement.

The bank of single-phase transformers or the common core three-phase transformer can be connected in one of four combinations relative to the primary and secondary connections.

Wye-Delta: Commonly used in a step-down transformer, wye connection on the HV side reduces insulation costs, the neutral point on the HV side can be grounded, stable with respect to unbalanced loads.

Delta-Wye: Commonly used in a step-up transformer for the same reasons as above.

Delta-Delta: Offers the advantage that one of the transformers can be removed while the remaining two transformers can deliver three-phase power at 58% of the original bank.

Wye-Wye: Rarely used, problems with unbalanced loads.
Wye-Delta Connection

\[ V \quad \frac{V}{\sqrt{3}} \quad N_1 \]
\[ I \quad \frac{I}{\sqrt{3}} \]

\[ \frac{\sqrt{3}aI}{a} \quad V \]

Delta-Wye Connection

\[ V \quad \frac{V}{a} \quad N_1 \]
\[ I \quad \frac{I}{\sqrt{3}} \]

\[ \frac{\sqrt{3}V}{a} \quad aI \]
\[ \frac{V}{\sqrt{3}} \]
Delta-Delta Connection

![Diagram of Delta-Delta Connection]

Wye-Wye Connection

![Diagram of Wye-Wye Connection]
Note that the voltage across a wye-connected primary or secondary winding is the line-to-neutral voltage while the voltage across a delta-connected primary or secondary winding is the line-to-line voltage. The magnitude of the complex three-phase power into or out of a three-phase transformer in a balanced system may be written as

\[ S = 3 V_w I_w \]

where \( V_w \) is the magnitude of the voltage across each winding and \( I_w \) is the magnitude of the current through each winding. In the wye-configuration, the winding voltage is the line-to-neutral voltage \( (V_{LN}) \) while the winding current is the line current \( (I_L) \). In the delta-configuration, the winding voltage is the line-to-line voltage \( (V_{LL}) \) while the winding current is the delta current \( (I_\Delta) \). Thus, for a wye-connected winding, the magnitude of the complex power is

\[ S_y = 3 V_{LN} I_L = 3 \frac{V_{LL}}{\sqrt{3}} I_L = \sqrt{3} V_{LL} I_L \]

The magnitude of the complex power for the delta-connected winding is

\[ S_\Delta = 3 V_{LL} I_\Delta = 3 V_{LL} \frac{I_L}{\sqrt{3}} = \sqrt{3} V_{LL} I_L \]

so that the equation for the complex power for either transformer winding connection is the same given the line-to-line voltage and the line current.

\[ S = \sqrt{3} V_{LL} I_L \]
PER-PHASE ANALYSIS OF THREE-PHASE TRANSFORMERS

Assuming the three transformers in the three-phase transformer are identical and the sources and loads in the three-phase problem are balanced, circuits involving a the three-phase transformer can be analyzed on a per-phase basis as illustrated in our study of three-phase circuits. As previously discussed, the easiest three-phase topology to analyze is the wye-wye connection. Thus, given any other configuration for the three-phase transformer other than wye-wye, one should transform the circuit into wye-wye form.

The equivalent turns ratio for the transformed wye-wye per-phase equivalent circuit for the transformer is the ratio of the primary line-to-line voltage to the secondary line-to-line voltage for the original configuration.

\[ a' = \frac{\text{Primary line-to-line voltage}}{\text{Secondary line-to-line voltage}} \]

The concept of the equivalent turns ratio can be illustrated by an example transformation of a transformer configuration.

The wye-delta and delta-wye configurations of three-phase transformers result in 30° phase shifts between the primary and secondary line-to-line voltages. The industry standard is such that the lower voltages in these configurations should lag the higher voltages by 30°. The wye-wye or delta-delta configurations produce line-to-line voltages in the primary and secondary that are in phase.
Example

Transform a wye-delta three-phase transformer into the wye-wye configuration and determine the equivalent turns ratio \( a' \) of the resulting wye-wye transformer. Draw the per-phase equivalent circuit for the resulting wye-wye transformer.

The line-to-neutral voltages across the windings of the equivalent wye-connected secondary are found by dividing the line-to-line voltages across the windings of the delta-connected secondary by \( \sqrt{3} \).

\[
V_{\Delta, LL} = \frac{V}{\sqrt{3}a}
\]

\[
V_{Y, LN} = \frac{V_{\Delta, LL}}{\sqrt{3}} = \frac{V}{3a}
\]
Comparing the voltages and currents of the primary and secondary windings, we see that the equivalent turns ratio of the wye-wye configuration is

$$a' = \frac{N_1}{N_2'} = \sqrt{3}a = \frac{\sqrt{3}N_1}{N_2} \quad \text{(Wye–Delta)}$$

The equivalent wye-wye model for the wye-delta connected three-phase transformer is

\begin{equation*}
\begin{array}{c}
\text{In a similar fashion, if we consider the transformation of the delta-wye and delta-delta configurations to the wye-wye configurations, we find equivalent turns ratios of}
\end{array}
\end{equation*}

$$a' = \frac{N_1'}{N_2} = \frac{a}{\sqrt{3}} = \frac{N_1}{\sqrt{3}N_2} \quad \text{(Delta–Wye)}$$

$$a' = \frac{N_1'}{N_2'} = a = \frac{N_1}{N_2} \quad \text{(Delta–Delta)}$$
Example (Per-phase equivalent circuit / three-phase transformer)

Three single-phase 50 kVA, 2300/230 V 60 Hz transformers are connected to form a three-phase 4000/230 V transformer bank (these voltages are line to line) which supplies a 120 kVA, 230 V, three-phase load with a power factor of 0.85 lagging. The equivalent impedance for each transformer referred to the LV winding is $(0.012 + j 0.016) \, \Omega$.

(a.) Determine the transformer configuration required and draw the per-phase equivalent circuit.
(b.) Determine the transformer winding currents.
(c.) Determine the primary voltage required to produce the rated output.
(d.) Determine the voltage regulation.

(a.) Individual transformers:
- Primary winding rated voltage $V_{1,\text{rated}} = 2300 \, V$
- Secondary winding rated voltage $V_{2,\text{rated}} = 230 \, V$
- Turns ratio $a = \frac{N_1}{N_2} = \frac{V_{1,\text{rated}}}{V_{2,\text{rated}}} = \frac{2300}{230} = 10$

3φ transformer:
- Primary line to line voltage $V_{1,\text{LL}} = 4000 \, V \approx 2300 \times \sqrt{3}$
- Secondary line to line voltage $V_{2,\text{LL}} = 230 \, V$
- Y-Y equivalent turns ratio $a' = \frac{V_{1,\text{LL}}}{V_{2,\text{LL}}} = \frac{4000}{230} = 17.39$

The required transformer connection is Wye-Delta.
The given equivalent impedance for each transformer is referred to the LV winding (secondary). This impedance referred to the HV input winding is

\[ Z_{eq1} = \alpha^2 Z_{eq2} = 10^2 (0.012 + j0.016) = (1.2 + j1.6) \, \Omega \]

Note that the turns ratio of the individual transformer is used to reflect the impedance between the primary and the secondary. The resulting wye-wye per-phase equivalent circuit is shown below.

(b.) The line current delivered to the three-phase load can be found from the complex power equation:

\[ S = \sqrt{3} V_{LL} I_L \quad \Rightarrow \quad I_L = \frac{S}{\sqrt{3} V_{LL}} = \frac{120000}{\sqrt{3} (230)} = 301.23 \, \text{A} \]

The actual current in the delta-connected secondary winding of this transformer is

\[ I_\Delta = \frac{I_L}{\sqrt{3}} = \frac{301.23}{\sqrt{3}} = 173.92 \, \text{A} \]

The corresponding current in the wye-connected primary is

\[ I_y = \frac{I_\Delta}{\alpha} = \frac{173.92}{10} = 17.39 \, \text{A} \]
(c.) To determine the line-to-line voltage on the primary required to produce a secondary line-to-line voltage of 230 V, we must analyze the per-phase equivalent circuit. In the per-phase equivalent circuit, the current $I_2$ is the secondary line current (magnitude = 301.23 A) while the voltage $V_2$ is the secondary line-to-neutral voltage (magnitude = $230/\sqrt{3} = 132.79$ V). The power factor of the load gives the phase angle difference between $V_2$ and $I_2$.

$$\text{PF} = 0.85 \text{ lagging} \quad \Rightarrow \quad (\theta_v - \theta_i)_{\text{Load}} = \cos^{-1}(0.85) = 31.79^\circ$$

Using the secondary line-to-neutral voltage as our reference gives

$$V_2 = 132.79 \angle 0^\circ \ V$$

$$I_2 = 301.23 \angle -31.79^\circ \ V$$

The secondary values can be reflected back to the primary according to the modified turns ratio $a'$.

$$V_2' = a'V_2 = (17.39)(132.79 \angle 0^\circ) = 2300 \angle 0^\circ \ V$$

$$I_2' = \frac{I_2}{a} = \frac{301.23 \angle -31.79^\circ}{17.39} = 17.39 \angle -31.79^\circ \ V$$
The resulting primary voltage \( V_1 \) (line-to-neutral) is

\[
V_1 = V_2' + I_2' Z_{eq1}
\]

\[
= 2300 \angle 0^\circ + (17.39 \angle -31.79^\circ)(2 \angle 53.13^\circ) \\
= 2300 \angle 0^\circ + 34.78 \angle 21.34^\circ \\
= 2332.4 \angle 0.31^\circ
\]

The magnitude of the primary line-to-line voltage is

\[
\sqrt{3} (2332.4) = 4039.8 \text{ V}
\]

(d.) The voltage regulation of this transformer is given by

\[
VR = \frac{|V_1| - |V_2'|_{\text{rated}}}{|V_2'|_{\text{rated}}} \times 100 \\
= \frac{2332.4 - 2300}{2300} \times 100 \\
= 1.41 \%
\]