Exam I

Instructions:

1. This test is closed book, closed notes, closed neighbor.

2. You may use a calculator. You may not use a computer, PDA, cell phone, or any wireless device.

3. There are 5 problems, all of which are equally weighted.

4. Work the problems in the space provided. If you need additional space, use the back side of the previous page.

5. Show all your work and put a box around all final answers.

6. You have 50 minutes to work the exam.

7. This exam is conducted under the Honor Code: “As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do.” Sign below to indicate that you have read, understood, and complied with these instructions and the Honor Code.

Signature: _________________________________
1. Consider the circuit below.

(a) Find $i_1$, $i_2$, and $v$.

KCL at ①: $-i_1 + 7 - 2 = 0 \Rightarrow i_1 = 7 - 2 = \boxed{5 \text{A}}$

KCL at ②: $i_1 - 7 - i_2 = 0 \Rightarrow i_2 = i_1 - 7 = 5 - 7 = \boxed{-2 \text{A}}$

KVL around loop ⑤: 
$$3 + v - v_A + (-4) = 0$$
$$\Rightarrow v = 3 + 8 + 4 = \boxed{9 \text{V}}$$

(b) Find the powers, $p_A$ and $p_B$, dissipated by circuit elements $A$ and $B$, respectively. Indicate whether circuit elements $A$ and $B$ are generating or consuming power.

$$p_A = v_A i_2 = 8 \cdot (-2) = \boxed{-16 \text{W}}$$

Since $p_A < 0$, $A$ is generating power.

$$p_B = 3 \cdot 2 = \boxed{6 \text{W}}$$

Since $p_B > 0$, $B$ is consuming power.
2. Using mesh analysis, find \( v \).

\[
\begin{align*}
\text{KVL around } i_1: \\
10i_1 + 12 + 10(i_1 - i_2) + 2i_1 + 8i_1 - 6 &= 0 \\
30i_1 - 10i_2 &= -6 & \text{(1)}
\end{align*}
\]

\[
\begin{align*}
\text{KVL around } i_2: \\
5i_2 + 5i_2 + 10(i_2 - i_1) - 12 &= 0 \\
-10i_1 + 20i_2 &= 12 & \text{(2)}
\end{align*}
\]

Solve (1) and (2):

\[ i_1 = 0 \text{ A} \]

\[ i_2 = \frac{3}{5} \text{ A} \]

\[ \therefore \quad v = 5i_2 = 5 \cdot \frac{3}{5} = 3 \text{ V} \]
3. Using nodal analysis, find $i$.

\[ \text{KCL at } V_1: \]
\[ 15 + \frac{0 - V_1}{2} + \frac{V_2 - V_1}{3} - 2i_1 = 0 \]
\[ 90 - 3V_1 + 2V_2 - 2V_1 - 12\left(\frac{V_1}{2}\right) = 0 \]
\[ -11V_1 + 2V_2 = -90 \quad (1) \]

\[ \text{KCL at } V_2: \]
\[ 2i_1 + \frac{V_1 - V_2}{3} + \frac{0 - V_2}{4} = 0 \]
\[ 24\left(\frac{V_1}{2}\right) + 4V_1 - 2V_2 - 3V_2 = 0 \]
\[ 16V_1 - 7V_2 = 0 \quad (2) \]

Solve (1) and (2):
\[ i = 14 \text{V} \]
\[ V_2 = 32 \text{V} \]

\[ \therefore i = \frac{V_2}{4} = \frac{32}{4} = 8 \text{A} \]
4. Find \( v \) using superposition by expressing \( v \) as

\[
v = v_1 + v_2
\]

where \( v_1 \) is due to the voltage source and \( v_2 \) is due to the current source.

\[
\begin{array}{c}
\text{Find } v_1: \text{ Zero current source} \\
6V \quad + \quad 1\Omega \quad 2\Omega \quad v_1 \quad + \quad 2\Omega \quad v_1 \quad - \\
\text{voltage division: } v_1 = \frac{2}{1+2} \times 6 = 4V
\end{array}
\]

\[
\begin{array}{c}
\text{Find } v_2: \text{ Zero voltage source} \\
1\Omega \quad 2\Omega \quad + \quad 2\Omega \quad \frac{3A}{2} \quad -
\end{array}
\]

\[
v_2 = 3 \times (1||2) \\
= 3 \times \frac{1}{1 + \frac{1}{2}} = 3 \times \frac{1}{3/2} = 2V
\]

\[
\therefore \quad v = v_1 + v_2 = 4V + 2V = 6V
\]
5. Consider the circuit below.

(a) Find the Thévenin equivalent circuit seen by the load resistor \( R_L \).

\[
R_T = 16 + (20||80) = 16 + 16 = 32 \, \Omega
\]

\[
V_T = 1 \cdot (20||80) = 1 \cdot 16 = 16 \, V
\]

Thevenin equivalent circuit:

(b) What value of \( R_L \) will maximize the power dissipated by \( R_L \)?

To maximize power:

\[
R_L = R_T = 32 \, \Omega
\]