Exam II

Instructions:
1. This test is closed book, closed notes, closed neighbor.
2. You may use a calculator. You may not use a computer, PDA, cell phone, or any wireless device.
3. There are 5 problems, all of which are equally weighted.
4. Work the problems in the space provided. If you need additional space, use the back side of the previous page.
5. Show all your work and put a box around all final answers.
6. You have 50 minutes to work the exam.
7. This exam is conducted under the Honor Code: “As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do.”  Sign below to indicate that you have read, understood, and complied with these instructions and the Honor Code.

Signature: ________________________________
1. (a) Find the equivalent capacitance seen by the terminals in the circuit below.

\[ C_{\text{EQ}} = \frac{1}{\frac{1}{4} + \frac{1}{1} + \frac{1}{4}} = \frac{1}{\frac{3}{2}} = \frac{2}{3} \text{ F} \]

\[ C_1 = \frac{1}{2} + \frac{1}{2} = 1 \text{ F} \]

(b) Find the equivalent inductance seen by the terminals in the circuit below.

\[ L_{\text{EQ}} = 10 \| (7 + (6 \| (2+4))) \]

\[ = 10 \| (7 + 3) \]

\[ = 10 \| 10 \]

\[ = \frac{1}{\frac{1}{10} + \frac{1}{10}} = 15 \text{ H} \]
2. Consider the circuit below.

(a) Find $v(t)$.

**Phasor domain ($\omega = 4$):**

\[
10 \angle 0^\circ \quad \frac{3}{\sqrt{3-j}} = -j
\]

**Voltage division:**

\[
v = \frac{-j}{3-j} \cdot 10 \angle 0^\circ
\]

\[
\approx 3.162 \angle -71.6^\circ
\]

\[\therefore v(t) = 3.162 \cos(4t - 71.6^\circ) \text{ V}\]

(b) Find the energy stored in the capacitor, $w(t)$, at time $t = 0$.

\[
w(t) = \frac{1}{2} C v^2(t)
\]

\[
w(0) = w(t)\big|_{t=0} = \frac{1}{2} \cdot \frac{1}{4} \cdot (3.162 \cos(-71.6^\circ))^2 \approx 0.125 \text{ J}
\]
3. Find \( v(t) \) for the circuit below.

\[ \begin{align*}
1 \Omega & \quad 1H & \quad 2H \\
8\cos(2t)V & \quad 1/2F & \quad -v(t)
\end{align*} \]

**Phasor Domain (\( \omega = 2 \)):**

\[ \begin{align*}
1 & \quad j\omega L = j2 \\
1/2 & \quad j\omega C = j4 \\
870^\circ & \quad 1/2 = -j
\end{align*} \]

**KCL at \( V \):**

\[ \frac{8-V}{1+j2} + \frac{0-V}{-j} + \frac{0-V}{2+j4} = 0 \]

\[ \left[ \frac{1}{1+j2} + \frac{1}{j} - \frac{1}{2+j4} \right] V = \frac{-8}{1+j2} \]

\[ (-0.3-j0.4) V = \frac{-8}{1+j2} \]

\[ : \quad V = \frac{-8}{(1+j2)(-0.3-j0.4)} \]

\[ \approx 7.155 \angle -116.6^\circ \]

\[ : \quad v(t) = 7.155 \cos(2t - 116.6^\circ) \quad V \]
4. Find $i(t)$ for the circuit below.

\[ 20 \Omega  \quad 10 \Omega \quad i(t) \]

\[ 40 \cos(t - 17^\circ) \text{V} \]

\[ 5 \text{H} \quad 0.1 \text{F} \]

**Phasor Domain ($\omega = 1$):**

\[ 40 \angle -17^\circ \quad 20 \quad 10 \]

\[ j\omega L = j5 \quad \text{I}\]

\[ j\omega C = \frac{1}{j\omega 10} = -j0 \]

**KVL I:**

\[ 20 \text{I}_1 + j5(\text{I}_1 - \text{I}_2) - 40 \angle -17^\circ = 0 \]

\[ (20 + j5) \text{I}_1 - j5 \text{I}_2 = 40 \angle -17^\circ \]

**KVL II:**

\[ 10 \text{I}_2 - j10 \text{I}_2 + j5(\text{I}_2 - \text{I}_1) = 0 \]

\[ -j5 \text{I}_1 + (10 - j5) \text{I}_2 = 0 \]

\[
\begin{bmatrix}
20 + j5 & -j5 \\
-j5 & 10 - j5
\end{bmatrix}
\begin{bmatrix}
\text{I}_1 \\
\text{I}_2
\end{bmatrix}
=
\begin{bmatrix}
40 \angle -17^\circ \\
0
\end{bmatrix}
\]

\[ \text{I}_1 = 1.754 \angle -32.3^\circ \]

\[ \text{I}_2 = 0.784 \angle 84.3^\circ \]

\[ i(t) = 0.784 \cos(t + 84.3^\circ) \text{A} \]
5. Find the phasor-domain Thévenin equivalent circuit for the circuit below as seen from the terminals A and B.

\[ 12 \cos(2t) \text{ V} \]

\[ 2 \Omega \]

\[ \frac{1}{4} \text{ F} \]

\[ 3 \text{ H} \]

\[ j \omega L = j \cdot 2 \cdot 3 = j 6 \]

\[ \frac{1}{j \omega C} = \frac{1}{j \cdot 2 \cdot \frac{1}{4}} = -j 2 \]

**Equivalent Impedance:**

\[ 2 \]

\[ \frac{3 \sqrt{6}}{2} \]

\[ -j8 \]

\[ Z_T = j611(2-j2) \]

\[ \frac{1}{\frac{1}{\sqrt{6}} + \frac{1}{2-j2}} \]

\[ \approx 3.795 \angle -18.40^\circ \]

**Thevenin Equivalent:**

\[ 16.1 \angle 26.6^\circ \]

\[ 3.795 \angle -18.40^\circ \]