Final Exam

Instructions:
1. This test is closed book, closed notes, closed neighbor.
2. You may use a calculator. You may not use a computer, PDA, cell phone, or any wireless device.
3. There are 8 problems, all of which are equally weighted.
4. Work the problems in the space provided. If you need additional space, use the back side of the previous page.
5. Show all your work and put a box around all final answers.
6. You have 3 hours to work the exam.
7. This exam is conducted under the Honor Code: “As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do.” Sign below to indicate that you have read, understood, and complied with these instructions and the Honor Code.

Signature: __________________________
1. Find $v_C(t)$ for $t > 0$.

\[
\begin{align*}
\text{\textbf{t} = 0^-} : & \quad v_C(0^-) = \frac{2}{2+2} \cdot 24 = 12V \\
& \quad v_C(0+) = V_C(0+) \\
\text{\textbf{KVL}} & \quad 2\dot{i}_1 + 2(\dot{v}_1 - \dot{v}_2) - 24 = 0 \\
& \quad 4\dot{i}_1 - 2\dot{v}_2 = 24 \\
\text{\textbf{KVL}} & \quad \dot{\dot{v}}_2 + 2\dot{v}_2 + 2(\dot{v}_2 - \dot{v}_1) = 0 \\
& \quad -2\ddot{v}_1 + 5\dot{v}_2 = 0 \\
& \begin{bmatrix} 4 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} \ddot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \end{bmatrix} \\
& \ddot{v}_1 = 7.5A \\
& \dot{v}_2 = 3A \\
\therefore v_C(\infty) &= 2\ddot{v}_2 = 6V
\end{align*}
\]

\[
\text{Time constant} : \quad \tau = \frac{1}{R_{eq}} = \frac{1}{3+2\parallel(1+1)} = \frac{1}{3+2\parallel2} = \frac{1}{3+1} = 0.5 \Omega
\]

\[
\therefore v_C(t) = [v_C(0^+) - v_C(\infty)] e^{-t/\tau} + v_C(\infty) \\
= [12 - 6] e^{-t/0.5} + 6 \\
= 6 e^{-t/2} + 6 \text{V}
\]
2. Consider the circuit below with input $i(t)$ and output $v(t)$.

![Circuit Diagram]

(a) Find the frequency response, $H(j\omega)$.

Phasor domain:

$$\text{KVL } \text{II}_1: \quad \frac{j\omega}{4} \text{II}_1 + 4\text{II}_1 + j\omega^3 (\text{II}_1 - \text{II}) = 0$$

$$\left(\frac{j\omega}{4} + 4 + j\omega^3\right)\text{II}_1 - j\omega^3 \text{II} = 0$$

$$\left(4 + j\omega^4 - 3\omega^2\right)\frac{\text{V}}{4} + 3\omega^2 \text{II} = 0$$

$$\left(4 - 3\omega^2 + j\omega^4\right)\text{V} = -12\omega^2 \text{II}$$

\[\therefore H(j\omega) = \frac{\text{V}}{\text{II}} = \frac{-12\omega^2}{4 - 3\omega^2 + j\omega^4}\]

(b) Find the AC steady-state $v(t)$ if $i(t) = 5 \cos(20t - 10^\circ) \text{ A}$.

$$\text{II} = 5^\circ - 10^\circ \quad \omega = 20$$

$$H(j\omega)\big|_{\omega=20} = \frac{-12 \cdot 400}{4 - 3.400 + j80} = 4 \angle 3.8^\circ$$

$$V = \text{II} \cdot H(j\omega)\big|_{\omega=20} = 5^\circ - 10^\circ \cdot 4 \angle 3.8^\circ = 20^\circ - 6.2^\circ$$

\[\therefore v(t) = 20 \cos(20t - 6.2^\circ)\]
3. In the circuit below, find \( i \) using nodal analysis.

\[
\delta = \frac{42 - \nu}{2}
\]
\[
\delta_1 = \frac{\nu}{8}
\]

\[ KCL \text{ at } \nu: \]
\[
\frac{42 - \nu}{2} + \frac{0 - \nu}{8} - 2\delta_1 = 0
\]
\[
168 - 4\nu - \nu - 16\delta_1 = 0
\]
\[
-5\nu - 16\left(\frac{\nu}{8}\right) = -168
\]
\[
-7\nu = -168
\]
\[
\therefore \nu = \frac{168}{7} = 24\,\text{V}
\]

\[
\therefore i = \frac{42 - \nu}{2} = \frac{42 - 24}{2} = \boxed{9\,\text{A}}
\]
4. In the circuit below, find \( v \) using mesh analysis.

**KVL \( i_1 \):**

\[
4i_1 + 2(i_1 - i_2) - 14 = 0
\]

\[
6i_1 - 2i_2 = 14
\]

**KVL \( i_2 \):**

\[
-4 + 3i_2 + 2(i_2 - i_1) = 0
\]

\[
-2i_1 + 5i_2 = 4
\]

\[
\begin{bmatrix}
6 & -2 \\
-2 & 5
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix} =
\begin{bmatrix}
14 \\
4
\end{bmatrix}
\]

\[
i_1 = 3 \text{ A}
\]

\[
i_2 = 2 \text{ A}
\]

\[
\Rightarrow v = 3i_2 = 3 \times 2 = 6 \text{ V}
\]
5. Find $v$ in the circuit below.

\[ \begin{align*}
& \text{KCL at } V_-: \\
& \frac{8-V_-}{1} + \frac{V_- - V}{5} + 0 = 0 \\
& 40 - 5V_- + V = 0 \\
& -6V_- + 5 = -40 \\
& \therefore V_- = \frac{40 + V}{6} \\
& \quad \quad = V_+ \\
& \text{KCL at } V_+: \\
& \frac{0-V_+}{5} + \frac{V - V_+}{1} + 0 = 0 \\
& -V_+ + 5V - 5V_+ = 0 \\
& -6V_+ = -5V \\
& -6\left[\frac{40 + V}{6}\right] = -5V \\
& -40 - V = -5V \\
& \therefore -40 = -2V \\
& \therefore V = 10V
\end{align*} \]
6. Consider the filter below.

(a) Find the frequency response, $H(j\omega)$.

\[ Z = \frac{1}{j\omega + \frac{4}{\omega}} = \frac{4}{1 + j\omega^4} \]

(b) Is the filter lowpass, highpass, or bandpass?

\[ |H(j\omega)|_{\omega=\infty} = 0 \quad \Rightarrow \quad \text{lowpass} \]

(c) Find the cutoff frequency $\omega_0$ of the filter.

\[ \frac{1 + 16\omega^2}{2} = 2 \quad \Rightarrow \quad \omega_0 = \frac{1}{4} \text{ rad/s} \]
7. (a) Find the phasor-domain Thévenin equivalent for the circuit below as seen from the terminals a and b.

\[
Z_T = (\frac{j4}{3j+4j2}) = \frac{1}{-j4 + j4} = 4 \angle 36.9^\circ
\]

(b) Find AC steady-state \( v(t) \) when the circuit below is attached to the circuit above at the indicated terminals.

Voltage division:

\[
\begin{align*}
V &= \frac{1}{4\angle 36.9^\circ + 1} \cdot (7.16L - 46.6 - 2L0^\circ) \\
&= 1.23L - 90.4^\circ
\end{align*}
\]

\[v(t) = 1.23 \cos(2t - 90.4^\circ) \text{ V}\]
8. Consider the phasor-domain circuit below.

(a) Find phasors \( V \) and \( I \) when the load is \( Z = 5\angle 40^\circ \), and the source is \( V_s = 10\angle -30^\circ \).

\[
\text{KCL} \quad V_4: \quad \frac{V_s - V_4}{2} + \frac{0 - V_4}{2} + 0 = 0
\]
\[
\frac{jV_s - jV_4 - V_4}{2} = 0
\]
\[
(1+j)V_4 = jV_s
\]
\[
V_4 = \frac{j}{1+j} V_s = \frac{j}{1+j} \cdot 10\angle -30^\circ = 7.07\angle 15^\circ
\]

\[
\text{KCL} \quad V: \quad \frac{V_s - V_4}{4} + \frac{V - V_4}{4} + 0 = 0
\]
\[
\frac{V_s - V_4 + V - V_4}{4} = 0
\]
\[
V = 2V_4 - V_s = 2\cdot 7.07\angle 15^\circ - 10\angle -30^\circ
\]
\[
= 10\angle 60^\circ
\]

\[
\mathbb{I} = \frac{V}{Z} = \frac{10\angle 60^\circ}{5\angle 40^\circ} = 2\angle 20^\circ
\]

(b) Find the average power \( P \) consumed by the load \( Z \).

\[
P = \frac{|V| |\mathbb{I}|}{2} \cos(\angle Z) = \frac{10 \cdot 2}{2} \cos 40^\circ
\]
\[
= 7.66 \text{ W}
\]