3.24. This problem is somewhat vague in that it asks for the voltage across $R_4$ but does not give us the polarity for such voltage. We will assume that this voltage, $v_2$, has the polarity as shown below.

Examing the voltage sources, we see:

$$v_1 + v = 5V \quad \rightarrow \quad v = 5 - v_1$$
$$v_3 = v_2 + 70v = v_2 + 70(5 - v_1) = -70v_1 + v_2 + 350$$

KCL at node $v_1$:

$$\frac{5 - v_1}{2.2k} + \frac{v_2 - v_1}{6.8k} + \frac{v_3 - v_1}{1.8k} = 0$$
$$1530 - 306v_1 + 99v_2 - 99v_1 + 374v_3 - 374v_1 = 0$$
$$-779v_1 + 99v_2 + 374v_3 = -1530$$
$$-779v_1 + 99v_2 + 374(-70v_1 + v_2 + 350) = -1530$$
$$-26959v_1 + 473v_2 = -132430 \quad (1)$$

KCL at supernode $S$:

$$\frac{v_1 - v_2}{6.8k} + \frac{v_1 - v_3}{1.8k} + \frac{0 - v_2}{220} = 0$$
$$99v_1 - 99v_2 + 374v_1 - 374v_3 - 3060v_2 = 0$$
$$473v_1 - 3159v_2 - 374v_3 = 0$$
$$473v_1 - 3159v_2 - 374(-70v_1 + v_2 + 350) = 0$$
$$26653v_1 - 3533v_2 = 130900 \quad (2)$$

Equations (1) and (2) are the node equations. Solving

$$\begin{bmatrix} -26959 & 473 \\ 26653 & -3533 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -132430 \\ 130900 \end{bmatrix}$$
yields

\[ v_1 \approx 4.912 \text{V} \]
\[ v_2 \approx 8.757 \text{mV} \]

Thus, we have \( v_2 \approx 8.757 \text{mV} \) as the desired answer.

3.27. We define three mesh currents \( i, i_1, \) and \( i_2 \) as shown below. We also define a loop \( S \) which avoids the dependent current source.

![Circuit Diagram](image)

We note:

\[ i - i_2 = 0.2v_x \quad \rightarrow \quad v_x = 5i - 5i_2 \]
\[ \frac{v_x}{1.2k} = i_1 - i_2 \quad \rightarrow \quad i_1 = \frac{v_x}{1.2k} + i_2 = \frac{5i - 5i_2}{1.2k} + i_2 = \frac{5}{1.2k}i + \frac{1195}{1.2k}i_2 \]

KVL around \( i_1 \):

\[
50i_1 + v_x - 5.6 = 0
\]
\[
50 \left( \frac{5}{1.2k}i + \frac{1195}{1.2k}i_2 \right) + (5i - 5i_2) = 5.6
\]
\[
\frac{6250}{1.2k}i + \frac{53750}{1.2k}i_2 = 5.6 \quad (3)
\]

KVL around loop \( S \):

\[
330i_2 + 440i - v_x = 0
\]
\[
330i_2 + 440i - (5i - 5i_2) = 0
\]
\[
435i + 335i_2 = 0 \quad (4)
\]

Equations (3) and (4) are the mesh equations. We solve

\[
\begin{bmatrix}
6250 \\ 1.2k \\
435 \\
335 \\
\end{bmatrix}
\begin{bmatrix}
i \\
i_2 \\
\end{bmatrix}
=
\begin{bmatrix}
5.6 \\
0 \\
\end{bmatrix}
\]

to yield

\[ i \approx -0.106 \text{A} \]
\[ i_2 \approx 0.137 \text{A} \]

Thus, we have \( i \approx -0.106 \text{A} \) as the desired answer.
3.41. The problem asks for the voltage across $R$, but, again, does not specify the polarity of said voltage. We denote this voltage $v$ and assume the polarity as indicated below:

![Circuit Diagram]

Working superposition, we consider each of the two sources in turn, zeroing out the other.

**Voltage Source** We zero out the current source by replacing it with an open circuit and find $v_1$:

![Redrawn Circuit Diagram]

We see that voltage division will directly yield $v_1$:

$$v_1 = \frac{0.23 \parallel 1}{0.23 \parallel 1 + 0.3} \cdot 12 \\ \approx 4.608 \text{V}$$

**Current Source** We zero out the voltage source by replacing it with a short circuit and find $v_2$:

![Redrawn Circuit Diagram]

Using current division coupled with Ohm’s law, we arrive at

$$v_2 = \frac{\frac{1}{0.23}}{1 + \frac{1}{0.3} + \frac{1}{0.23}} \cdot 12 \cdot 0.23 \\ \approx 1.382 \text{V}$$
Thus, we have by superposition

\[ v = v_1 + v_2 \]
\[ \approx 4.608 + 1.382 \]
\[ = 5.99V \]

3.51. To find the Thevenin equivalent circuit, we first find the equivalent resistance, \( R_T \), with the voltage source zeroed out (i.e., short circuit):

Thus,

\[ R_T = 1 + 5 \parallel 4 \]
\[ \approx 3.222\Omega \]

Next, we find the equivalent, open-circuit voltage, \( v_T \):

Using voltage division, we have

\[ v_T = \frac{4}{4+5} \cdot 36 \]  \hspace{1cm} (5)
\[ = 16V \]  \hspace{1cm} (6)

Thus, the Thevenin equivalent circuit is:
To find the Thevenin equivalent circuit, we first find the equivalent resistance, \( R_T \), with the voltage source zeroed out (i.e., short circuit) and the current source zeroed out (i.e., open circuit):

\[
\begin{align*}
R_T &= 4\Omega \\
\end{align*}
\]

We have directly

\( R_T = 4\Omega \)

We then find the open-circuit voltage, \( v_T \):

\[
\begin{align*}
\text{KVL around loop 1:} & \\
-3 + v_T + v_1 &= 0 \\
\therefore \quad v_T &= 3 - v_1 \\
&= 3 - 8 \\
&= -5\text{V} \\
\end{align*}
\]

Thus, the Thevenin equivalent circuit, with the 3-\( \Omega \) load resistor is:

\[
\begin{align*}
\text{We employ voltage division to find } v: \\
&= \frac{3}{3 + 4} (-5) \\
&= -2.143\text{V}
\end{align*}
\]
3.53. To find the Norton equivalent circuit, we first find the equivalent resistance, $R_N$, with the voltage source zeroed out (i.e., short circuit) and the current source zeroed out (i.e., open circuit):

$$R_N = 1 + 3 + 1 \parallel 3$$

$$= 4 + \frac{1}{1 + \frac{3}{3}}$$

$$= 4.75\Omega$$

We then find the short-circuit current, $i_N$:

We note:

$$i_2 - i_N = 2 \quad \rightarrow \quad i_2 = i_N + 2$$

KVL around $i_1$:

$$i_1 + 3(i_1 - i_2) - 2 = 0$$

$$4i_1 - 3i_2 = 2$$

$$4i_1 - 3(i_N + 2) = 2$$

$$4i_1 - 3i_N = 8$$

(7)

KVL around loop S:

$$i_2 + 3i_N + 3(i_2 - i_1) = 0$$

$$4i_2 - 3i_1 + 3i_N = 0$$

$$4(i_N + 2) - 3i_1 + 3i_N = 0$$

$$-3i_1 + 7i_n = -8$$

(8)

Equations (7) and (8) are the mesh equations. We solve

$$\begin{bmatrix} 4 & -3 \\ -3 & 7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_N \end{bmatrix} = \begin{bmatrix} 8 \\ -8 \end{bmatrix}$$

and arrive at

$$i_1 \approx 1.684A$$

$$i_N \approx -0.421A$$
The Norton equivalent circuit is then:

\[ -0.421 \, \text{A} \quad 4.75 \, \Omega \]