5.33. **Step 1:** steady state for $t = 0^-$:

In steady state, the inductor is a short circuit, and the circuit appears as:

![Circuit Diagram 1]

The inductor current is found by KVL:

$$13i_L(0^-) + 20 - 23 + 0.7i_L(0^-) = 0$$

$$13.7i_L(0^-) = 3$$

$$i_L(0^-) \approx 0.219 \text{ A}$$

Since, inductor current cannot change instantaneously:

$$i_L(0^+) = i_L(0^-) \approx 0.219 \text{ A}$$

**Step 2:** steady state for $t = \infty$:

At $t = \infty$, the transient dies out and the circuit is in steady state. Thus, the inductor is again equivalent to a short circuit, and the circuit becomes:

![Circuit Diagram 2]

KVL around $i_L(\infty)$:

$$13i_L(\infty) + 20 + 330k_i_L(\infty) = 0$$

$$i_L(\infty) \approx -60.6 \mu \text{A}$$

**Step 3:** time constant for $t > 0$:

For $t > 0$, the circuit becomes:
This is a first-order $RL$ circuit. The time constant is

$$\tau = \frac{L}{R_{EQ}} = \frac{23\,\text{mH}}{13\,\text{kΩ} + 330\,\text{kΩ}} \approx 69.7\,\text{ns}$$

where $R_{EQ}$ is the equivalent resistance as seen by the inductor, which is equal to the two resistors in series.

**Step 4:** The inductor current for $t > 0$ is

$$i_L(t) = \left[ i_L(0^+) - i_L(\infty) \right] e^{-t/\tau} + i_L(\infty)$$

$$\approx [0.219 - (-60.6\,\mu\text{A})] e^{-t/69.7\,\text{ns}} + (-60.6\,\mu\text{A})$$

$$\approx 0.22 e^{-t/69.7 \times 10^{-9}} - 60.6 \times 10^{-6}\,\text{A}$$

**Step 5:** We note that $i_{R_3}(t) = -i_L(t)$, so

$$i_{R_3}(t) = 60.6 \times 10^{-6} - 0.22 e^{-t/69.7 \times 10^{-9}}\,\text{A}$$

5.34. **Step 1:** steady state for $t = 0^-$:

In steady state, the capacitor is an open circuit, and the circuit appears as:

Since no current is flowing in the 13-kΩ resistor, we have

$$v_C(0^-) = 17\,\text{V}.$$ 

Since, capacitor voltage cannot change instantaneously:

$$v_C(0^+) = v_C(0^-) = 17\,\text{V}.$$ 

**Step 2:** steady state for $t = \infty$:

At $t = \infty$, the transient dies out and the circuit is in steady state. Thus, the capacitor is again equivalent to an open circuit, and the circuit becomes:
Again, because no current flows through the 13-kΩ resistor, we have
\[ v_C(\infty) = v(\infty), \]
and it is clear that \( v(\infty) \) can be obtained directly from voltage division:
\[ v_C(\infty) = v(\infty) \]
\[ = \frac{14k}{14k + 14k} \cdot 11 \]
\[ = 5.5 \text{ V} \]

**Step 3:** time constant for \( t > 0 \):
For \( t > 0 \), the circuit becomes:

This is a first-order \( RC \) circuit. The equivalent resistance seen by the capacitor is:

\[ R_{\text{EQ}} = 13k + 14k \parallel 14k \]
\[ = 20 \text{ kΩ} \]

The time constant then is
\[ \tau = R_{\text{EQ}}C \]
\[ = 20k \cdot 70\text{n} \]
\[ = 1.4 \text{ ms} \]
Step 4: The capacitor voltage for \( t > 0 \) is

\[
v_C(t) = [v_C(0^+) - v_C(\infty)] e^{-t/\tau} + v_C(\infty)
\]

\[
= [17 - 5.5] e^{-t/1.4n} + 5.5
\]

\[
= 11.5 e^{-t/1.4 \times 10^{-3}} + 5.5 \text{ V}
\]

Step 5: To find \( v(t) \), we use KCL at the top node in the circuit for \( t > 0 \); this node is \( v(t) \) itself:

\[
\frac{11 - v(t)}{14k} + \frac{v_C(t) - v(t)}{13k} + \frac{0 - v(t)}{14k} = 0
\]

\[
\left[\frac{1}{14k} - \frac{1}{13k} - \frac{1}{14k}\right] v(t) = -\frac{1}{13k} v_C(t) - \frac{11}{14k}
\]

\[
\approx 0.35 v_C(t) + 3.58
\]

\[
\approx 4.03 e^{-t/1.4 \times 10^{-3}} + 5.51 \text{ V}
\]

(a) \( v(t) = 4.03 e^{-t/1.4 \times 10^{-3}} + 5.51 \text{ V} \)

(b) The final value of \( v(t) \) is

\[ v(\infty) = 5.51 \text{ V}. \]

98% of \( v(0^+) - v(\infty) \) is

\[ 0.98 \cdot (9.54 - 5.51) \approx 3.94 \text{ V}. \]

So, we want to find the time at which

\[ v(t) = v(0^+) - 3.94 = 9.54 - 3.94 = 5.59 \text{ V}. \]

To find the time at which this voltage occurs, we solve

\[ 5.59 = 4.03 e^{-t/1.4 \times 10^{-3}} + 5.51 \]

to yield

\[ t \approx 5.49 \text{ ms}. \]

5.41. We first note that the two capacitors are in parallel and thus act as a single equivalent capacitance of

\[ C_{\text{EQ}} = 4 + 4 = 8 \text{ F}. \]

Combining the two capacitances into a single capacitance makes this circuit into a first-order \( RC \) circuit. Also, since \( S_1 \) is always open, \( R_1 \) and the 20-V source are permanently removed from the circuit and so will be disregarded.

Step 1: steady state for \( t = 0^- \):

In steady state, \( C_{\text{EQ}} \) is an open circuit. But \( S_2 \) is open, disconnecting the 4-A source from the capacitors. Thus,

\[ v_C(0^+) = v_C(0^-) = 0 \text{ V}. \]

Step 2: steady state for \( t = \infty \):

At \( t = \infty \), \( S_2 \) is closed, the transient dies out, and the circuit is in steady state. Thus, \( C_{\text{EQ}} \) is again equivalent to an open circuit, and the circuit becomes:

\[
\begin{array}{c}
4 \Omega \\
+ \ 3 \Omega \\
\downarrow \ 6 \Omega \\
\downarrow \\
4 \text{ A}
\end{array}
\]

We have

\[ v_C(\infty) = (3 || 6) \cdot 4 = 8 \text{ V} \]
Step 3: time constant for \( t > 0 \):
For \( t > 0 \), the circuit becomes:

\[
\begin{align*}
\text{\[4 \Omega\]} & \quad \text{\[8 \text{ F}\]} \quad \text{\[3 \Omega\]} \quad \text{\[6 \Omega\]} \\
\text{\[4 \text{ A}\]} & \quad \text{\[\text{v}_C(t)\]} \\
\end{align*}
\]

This is a first-order RC circuit. The equivalent resistance seen by the capacitor is:

\[
R_{\text{EQ}} = 4 + 3 \parallel 6 \\
= 6 \Omega
\]

The time constant then is

\[
\tau = R_{\text{EQ}} C_{\text{EQ}} \\
= 6 \cdot 8 \\
= 48 \text{ s}
\]

Step 4: The capacitor voltage for \( t > 0 \) is

\[
v_C(t) = \left[ v_C(0^+) - v_C(\infty) \right] e^{-t/\tau} + v_C(\infty) \\
= [0 - 8] e^{-t/48} + 8 \\
= -8e^{-t/48} + 8 \text{ V}
\]

(a) \( v_C(0^+) = 0 \text{ V} \)
(b) \( \tau = 48 \text{ s} \)

(c) \( v_C(t) = -8e^{-t/48} + 8 \text{ V} \) for \( t > 0 \). A plot of the function over the interval \( t \in [0, 300] \) is shown below:

This plot is generated with the following MATLAB code:
\( t = \text{linspace}(0, 300, 1000); \)
\( \text{vc} = -8*\text{exp}(-t/48) + 8; \)
\( \text{plot}(t, \text{vc}, 'k-', 'LineWidth', 2); \)
\( \text{xlabel('t (sec)');} \)
\( \text{ylabel('v_C(t) (V)');} \)

(d) The voltages at the specified times are:

<table>
<thead>
<tr>
<th>( t )</th>
<th>( v_C(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 V</td>
</tr>
<tr>
<td>( \tau )</td>
<td>5.067 V</td>
</tr>
<tr>
<td>2( \tau )</td>
<td>6.917 V</td>
</tr>
<tr>
<td>5( \tau )</td>
<td>7.946 V</td>
</tr>
<tr>
<td>10( \tau )</td>
<td>7.999 V</td>
</tr>
</tbody>
</table>

5.44. Again, the two capacitors in parallel are equivalent to a single 8-F capacitor \((C_{EQ})\).

**Step 1:** steady state for \( t = 0^- \):
In steady state, \( C_{EQ} \) is an open circuit. But both \( S_1 \) and \( S_2 \) are open, disconnecting the both sources from the capacitors. Thus, \( v_C(0^+) = v_C(0^-) = 0 \text{ V} \).

**Step 2:** steady state for \( t = \infty \):
At \( t = \infty \), both \( S_1 \) and \( S_2 \) are closed, the transient dies out, and the circuit is in steady state. Thus, \( C_{EQ} \) is again equivalent to an open circuit, and the circuit becomes:

To find \( v_C(\infty) \), we solve a nodal analysis:

**KVL at \( v_C \):**
\[
\frac{20 - v_C}{5} + \frac{v_1 - v_C}{4} = 0
\]
\[
-\frac{9}{20} v_C + \frac{1}{4} v_1 = -4
\] (1)

**KVL at \( v_1 \):**
\[
\frac{v_C - v_1}{4} - \frac{1}{3} v_1 - \frac{1}{6} v_1 + 4 = 0
\]
\[
\frac{1}{4} v_C - \frac{3}{4} v_1 = -4
\] (2)

Solving (1) and (2) simultaneously yields:

\( v_C(\infty) \approx 14.5 \text{ V} \)
\( v_1(\infty) \approx 10.2 \text{ V} \)

**Step 3:** time constant for \( t > 0 \):
For \( t > 0 \), the circuit becomes:
This is a first-order $RC$ circuit. The equivalent resistance seen by the capacitor is:

$$R_{EQ} = (4 + (3 \parallel 6)) \parallel 5$$
$$= 6 \parallel 5$$
$$\approx 2.727 \Omega$$

The time constant then is

$$\tau = R_{EQ}C_{EQ}$$
$$\approx 2.727 \cdot 8$$
$$= 21.82 \text{ s}$$

Step 4: The capacitor voltage for $t > 0$ is

$$v_C(t) = [v_C(0^+) - v_C(\infty)] e^{-t/\tau} + v_C(\infty)$$
$$= [0 - 14.5] e^{-t/21.8} + 14.5$$
$$= -14.5 e^{-t/21.8} + 14.5 \text{ V}$$

(a) $v_C(0^+) = 0 \text{ V}$
(b) $\tau = 21.8 \text{ s}$
(c) $v_C(t) = -14.5 e^{-t/21.8} + 14.5 \text{ V}$ for $t > 0$. A plot of the function over the interval $t \in [0, 300]$ is shown below:
This plot is generated with the following MATLAB code:

```matlab
  t = linspace(0, 300, 1000);
  vc = -14.5*exp(-t/21.8) + 14.5;
  plot(t, vc, 'k-', 'LineWidth', 2);
  xlabel('t (sec)');
  ylabel('v_C(t) (V)');
```

(d) The voltages at the specified times are:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$v_C(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 V</td>
</tr>
<tr>
<td>$\tau$</td>
<td>9.166 V</td>
</tr>
<tr>
<td>2$\tau$</td>
<td>12.538 V</td>
</tr>
<tr>
<td>5$\tau$</td>
<td>14.402 V</td>
</tr>
<tr>
<td>10$\tau$</td>
<td>14.499 V</td>
</tr>
</tbody>
</table>

5.49. **Step 1:** steady state for $t = 0^-$:

In steady state, the inductor is a short circuit, and we have:

The current from the voltage source is

$$i_1(0^-) = \frac{100}{1000 + 5 \parallel 2.5}$$

$$\approx 99.8 \text{ mA}$$

From current division:

$$i_L(0^-) = \frac{1}{\frac{1}{5} + \frac{1}{2.5}} i_1(0^-)$$

$$\approx 66.6 \text{ mA}$$

Since the inductor current cannot change instantaneously,

$$i_L(0^+) = i_L(0^-) \approx 66.6 \text{ mA}.$$  

**Step 2:** steady state for $t = \infty$:

At $t = \infty$, the 1000-\Omega resistor has been replaced with a 10-\Omega resistor, the transient dies out, and the circuit is in steady state. Thus, the inductor is again equivalent to a short circuit, and the circuit becomes:
The current from the voltage source is

\[ i_1(\infty) = \frac{100}{10 + 5 \parallel 2.5} \approx 8.57 \text{ A} \]

From current division:

\[ i_L(\infty) = \frac{\frac{1}{2.5}}{\frac{1}{5} + \frac{1}{2.5}} i_1(\infty) \approx 5.71 \text{ A} \]

**Step 3:** time constant for \( t > 0 \):

For \( t > 0 \), the circuit becomes:

This is a first-order RL circuit. The equivalent resistance seen by the inductor is:

\[ R_{\text{EQ}} = 2.5 + 10 \parallel 5 \approx 5.83 \Omega \]

The time constant then is

\[ \tau = \frac{L}{R_{\text{EQ}}} \approx \frac{0.1}{5.83} \approx 17.2 \text{ ms} \]

**Step 4:** The inductor current for \( t > 0 \) is

\[ i_L(t) = [i_L(0^+) - i_L(\infty)] e^{-t/\tau} + i_L(\infty) \approx [0.0666 - 5.71] e^{-t/0.0172} + 5.71 \approx -5.64e^{-t/0.0172} + 5.71 \text{ A} \]
The plot of $i_L(t)$ for $t \in [0, 0.1]$ is shown below.

![Plot of $i_L(t)$](image)

This plot is produced by the following MATLAB code:

```matlab
t = linspace(0, 0.1, 1000);
vc = -5.64*exp(-t/0.0172) + 5.71;
plot(t, vc, 'k-', 'LineWidth', 2);
xlabel('t (sec)');
ylabel('i_L(t) (A)');
```

The time that $i_L(t) = 5$ A is

\[
5 = -5.64e^{-t/0.0172} + 5.71
\]

\[
t \approx 35.6 \text{ ms}
\]