Assignment (due Apr. 27)

1. A linear, time-invariant, causal discrete-time system is described by the difference equation

\[ y[n + 2] + y[n] = 2x[n + 1] - x[n], \]

where \( x[n] \) is the system input and \( y[n] \) is the system output.

(a) Find \( h[n] \), the impulse response of the system.

(b) The step response, \( g[n] \), of a system is the output when the input is \( u[n] \) and zero initial energy is present in the system. Find the step response of the system above.

(c) Suppose the input to the system is \( x[n] = 2^n u[n] \), and the initial conditions on the system output are

\[ y[0] = -2, \quad y[1] = -1. \]

Find the output \( y[n] \).

(d) Suppose the input to the system is \( x[n] = 2^n u[n] \), and the initial conditions on the system output are

\[ y[n] = 0, \quad n < 0. \]

Find the output \( y[n] \).

2. Consider the discrete-time system shown below (\( \Delta \) indicates the delay operator).

(a) Determine the transfer function \( H(z) \) of the system.

(b) Determine the difference equation for the system.

(c) Compute the output response \( y[n] \) when \( x[n] = 4u[n] \) assuming no initial energy in the system.