1. (a) $x(t)$ is sampled with sampling period $T = \frac{\pi}{15}$ sec. to produce $x_s(t)$. In the frequency domain, we have

$$X_s(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s),$$

where the sampling frequency is

$$\omega_s = \frac{2\pi}{T} = 30 \text{ rad/s}.$$

We note that the bandwidth of $x(t)$ is $B = 10 \text{ rad/s}$, and the sampling frequency satisfies the Nyquist sampling criterion of

$$\omega_s = 30 \geq 2B = 2 \cdot 10 = 20 \text{ rad/s},$$

so there will be no aliasing.

The spectrum of $x(t)$ is

![X(\omega)](image)

The spectrum of $x_s(t)$ is

![X_s(\omega)](image)

(b) Now the sampling period is

$$T = \frac{2\pi}{15} \text{ sec.}$$

so that the sampling frequency is

$$\omega_s = 15 \text{ rad/s}.$$

Since

$$\omega_s = 15 < 2B = 20,$$

aliasing will occur.

The spectrum of $x_s(t)$ is

![X_s(\omega)](image)
2. The amplitude spectrum of $x(t)$ is

![Amplitude Spectrum of x(t)](image)

The bandwidth of $x(t)$ is $B = 2$ rad/s.

(a) For a sampling period of

$$T = \frac{\pi}{4} \text{s},$$

the sampling frequency is

$$\omega_s = \frac{2\pi}{T} = 8 \text{ rad/s}.$$

Since

$$\omega_s = 8 \geq 2B = 4,$$

no aliasing will occur.

The spectrum of $x_s(t)$ is

![Spectrum of x_s(t)](image)

(b) For a sampling period of

$$T = \frac{\pi}{2} \text{s},$$

the sampling frequency is

$$\omega_s = \frac{2\pi}{T} = 4 \text{ rad/s}.$$

Since

$$\omega_s = 4 \geq 2B = 4,$$

no aliasing will occur.

The spectrum of $x_s(t)$ is

![Spectrum of x_s(t)](image)
(c) For a sampling period of 
\[ T = \frac{2\pi}{3} \text{ s}, \]
the sampling frequency is 
\[ \omega_s = \frac{2\pi}{T} = 3 \text{ rad/s}. \]

Since \( \omega_s = 3 < 2B = 4 \), aliasing will occur.

The spectrum of \( x_s(t) \) is

3. (a) The input signal to the sampler is 
\[ x(t) = 1 + \cos(15\pi t), \]
which, in the frequency domain, is 
\[ X(\omega) = 2\pi \delta(\omega) + \pi [\delta(\omega + 15\pi) + \delta(\omega - 15\pi)]. \]

The spectrum for \( x(t) \) is

The bandwidth of \( x(t) \) is \( B = 15\pi \text{ rad/s}. \)

When the sampler is operating with sampling period 
\[ T = 0.1 \text{ s}, \]
the sampling frequency is 
\[ \omega_s = \frac{2\pi}{T} = 20\pi \text{ rad/s}. \]

Since \( \omega_s = 20\pi < 2B = 2 \cdot 15\pi = 30\pi \), aliasing will occur.

The spectrum of the sampled signal, \( x_s(t) \), is
The ideal lowpass reconstruction filter is

\[
H(\omega) = \begin{cases} 
T, & -\frac{\omega_s}{2} \leq \omega \leq \frac{\omega_s}{2}, \\
0, & \text{else},
\end{cases}
\]

\[
= \begin{cases} 
\frac{1}{10}, & -10\pi \leq \omega \leq 10\pi, \\
0, & \text{else},
\end{cases}
\]

The spectrum, \(Y(\omega)\), of signal at the output of the reconstruction filter, \(y(t)\), is thus

\[
Y(\omega) \approx 1 + \cos(5\pi t),
\]

which is not equal to the input \(x(t)\) due to aliasing. It is interesting to note that the cosine of frequency \(15\pi\) rad/s has been “aliased” into a cosine of frequency \(5\pi\) rad/s.

(b) The input to the sampler is, in the frequency domain,

\[
X(\omega) = \frac{1}{1 + j\omega}.
\]

This signal is not bandlimited, since \(X(\omega)\) is nonzero for all \(\omega\). Consequently, our sampling/reconstruction system will exhibit aliasing no matter how large the sampling frequency is.

When the sampling period is

\[
T = 1\text{s},
\]

the sampling frequency is

\[
\omega_s = 2\pi\text{rad/s}.
\]

The magnitude of the sampled signal, \(|X_s(\omega)|\) is shown below. The sampled spectrum is

\[
X_s(\omega) = \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s).
\]

For the plot below, we approximate the infinite sum with a truncated sum, \(-100 \leq k \leq 100\).

```matlab
w = linspace(-20, 20, 1000);
T = 1;
w_s = 2*pi/T;
Xs = zeros(1, length(w));
clf
hold on
for k = -100:100
    Xk = 1 ./ (j*(w - k*w_s) + 1);
    Xs = Xs + Xk;
    plot(w, abs(Xk), 'k--');
end
plot(w, abs(Xs), 'k-', 'LineWidth', 2);
xlabel('\omega (rad/s)');
ylabel('|X_s(\omega)|');
grid
```