1. (a) \( x(t) \) is sampled with sampling period \( T = \frac{\pi}{15} \) sec. to produce \( x_s(t) \). In the frequency domain, we have

\[
X_s(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s),
\]

where the sampling frequency is

\[
\omega_s = \frac{2\pi}{T} = 30 \text{ rad/s}.
\]

We note that the bandwidth of \( x(t) \) is \( B = 10 \text{ rad/s} \), and the sampling frequency satisfies the Nyquist sampling criterion of

\[
\omega_s = 30 \geq 2B = 2 \cdot 10 = 20 \text{ rad/s},
\]

so there will be no aliasing.

The spectrum of \( x(t) \) is

(b) Now the sampling period is

\[
T = \frac{2\pi}{15} \text{ sec.}
\]

so that the sampling frequency is

\[
\omega_s = 15 \text{ rad/s}.
\]

Since

\[
\omega_s = 15 < 2B = 20,
\]

aliasing will occur.

The spectrum of \( x_s(t) \) is
2. The amplitude spectrum of $x(t)$ is

\[ |X(\omega)| \]

The bandwidth of $x(t)$ is $B = 2$ rad/s.

(a) For a sampling period of $T = \frac{\pi}{4}$ s,

the sampling frequency is

\[ \omega_s = \frac{2\pi}{T} = \frac{2\pi}{\frac{\pi}{4}} = 8 \text{ rad/s}. \]

Since $\omega_s = 8 \geq 2B = 4$,

no aliasing will occur.

The spectrum of $x_s(t)$ is

(b) For a sampling period of $T = \frac{\pi}{2}$ s,

the sampling frequency is

\[ \omega_s = \frac{2\pi}{T} = \frac{2\pi}{\frac{\pi}{2}} = 4 \text{ rad/s}. \]

Since $\omega_s = 4 \geq 2B = 4$,

no aliasing will occur.

The spectrum of $x_s(t)$ is
(c) For a sampling period of 
\[ T = \frac{2\pi}{3} \text{ s}, \]
the sampling frequency is
\[ \omega_s = \frac{2\pi}{T} = 3 \text{ rad/s}. \]

Since \( \omega_s = 3 < 2B = 4, \) aliasing will occur.

The spectrum of \( x_s(t) \) is
\[
\begin{align*}
|X_s(\omega)| &= \\
&= \frac{\pi}{2\pi} \\
&= \frac{1}{2}
\end{align*}
\]

\[
\omega - \frac{35\pi}{30\pi} - \frac{30\pi}{25\pi} - \frac{20\pi}{15\pi} - \frac{15\pi}{10\pi} - \frac{10\pi}{5\pi} 0 \frac{5\pi}{2\pi} \frac{10\pi}{\pi} \frac{15\pi}{20\pi} \frac{20\pi}{25\pi} \frac{25\pi}{30\pi} \frac{30\pi}{35\pi}
\]

3. (a) The input signal to the sampler is
\[ x(t) = 1 + \cos(15\pi t), \]
which, in the frequency domain, is
\[ X(\omega) = 2\pi \delta(\omega) + \pi [\delta(\omega + 15\pi) + \delta(\omega - 15\pi)]. \]

The spectrum for \( x(t) \) is
\[
\begin{align*}
X(\omega) &= \frac{\pi}{2\pi} \\
&= \frac{1}{2}
\end{align*}
\]

\[
\omega - \frac{35\pi}{30\pi} - \frac{30\pi}{25\pi} - \frac{20\pi}{15\pi} - \frac{15\pi}{10\pi} - \frac{10\pi}{5\pi} 0 \frac{5\pi}{2\pi} \frac{10\pi}{\pi} \frac{15\pi}{20\pi} \frac{20\pi}{25\pi} \frac{25\pi}{30\pi} \frac{30\pi}{35\pi}
\]

The bandwidth of \( x(t) \) is \( B = 15\pi \text{ rad/s}. \)

When the sampler is operating with sampling period
\[ T = 0.1 \text{ s}, \]
the sampling frequency is
\[ \omega_s = \frac{2\pi}{T} = 20\pi \text{ rad/s}. \]

Since
\[ \omega_s = 20\pi < 2B = 2 \cdot 15\pi = 30\pi, \]
aliasing will occur.

The spectrum of the sampled signal, \( x_s(t) \), is
\[
\begin{align*}
X_s(\omega) &= \frac{\pi}{2\pi} \\
&= \frac{1}{2}
\end{align*}
\]

\[
\omega - \frac{35\pi}{30\pi} - \frac{30\pi}{25\pi} - \frac{20\pi}{15\pi} - \frac{15\pi}{10\pi} - \frac{10\pi}{5\pi} 0 \frac{5\pi}{2\pi} \frac{10\pi}{\pi} \frac{15\pi}{20\pi} \frac{20\pi}{25\pi} \frac{25\pi}{30\pi} \frac{30\pi}{35\pi}
\]
The ideal lowpass reconstruction filter is

\[
H(\omega) = \begin{cases} 
T, & -\frac{\omega_s}{2} \leq \omega \leq \frac{\omega_s}{2}, \\
0, & \text{else,}
\end{cases}
\]

\[
= \begin{cases} 
\frac{1}{10}, & -10\pi \leq \omega \leq 10\pi, \\
0, & \text{else,}
\end{cases}
\]

The spectrum, \(Y(\omega)\), of signal at the output of the reconstruction filter, \(y(t)\), is thus

\[
Y(\omega) = \frac{\pi}{\omega - 35\pi} \frac{\omega}{30\pi} \frac{\omega}{25\pi} \frac{\omega}{20\pi} \frac{\omega}{15\pi} \frac{\omega}{10\pi} \frac{\omega}{5\pi} 0 \frac{\omega}{5\pi} \frac{\omega}{10\pi} \frac{\omega}{15\pi} \frac{\omega}{20\pi} \frac{\omega}{25\pi} \frac{\omega}{30\pi} \frac{\omega}{35\pi}
\]

In the time domain, the output of the reconstruction filter is

\[y(t) = 1 + \cos(5\pi t),\]

which is not equal to the input \(x(t)\) due to aliasing. It is interesting to note that the cosine of frequency \(15\pi\) rad/s has been “aliased” into a cosine of frequency \(5\pi\) rad/s.

(b) The input to the sampler is, in the frequency domain,

\[X(\omega) = \frac{1}{1 + j\omega}.\]

This signal is not bandlimited, since \(X(\omega)\) is nonzero for all \(\omega\). Consequently, our sampling/reconstruction system will exhibit aliasing no matter how large the sampling frequency is.

When the sampling period is

\[T = 1 \text{ s},\]

the sampling frequency is

\[\omega_s = 2\pi \text{ rad/s}.
\]

The magnitude of the sampled signal, \(|X_s(\omega)|\), is shown below. The sampled spectrum is

\[X_s(\omega) = \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s).\]

For the plot below, we approximate the infinite sum with a truncated sum, \(-100 \leq k \leq 100\).