MOTION-COMPENSATED COMPRESSED-SENSING RECONSTRUCTION FOR DYNAMIC MRI

Sungkwang Mun and James E. Fowler
Department of Electrical and Computer Engineering, Geosystems Research Institute,
Mississippi State University, USA

ABSTRACT

Compressed-sensing reconstruction using motion estimation and compensation for dynamic MRI data is proposed. Reconstruction is driven from a residual in the k-space domain between the current-frame measurements and a corresponding motion-compensated prediction. Due to the periodicity commonly exhibited in dynamic MRI, a telescopic motion search through the entire group of pictures is used to determine the best match for the block-based motion estimation. Experimental comparisons demonstrate improved performance as compared to existing dynamic-MRI reconstructions, both those with and without motion compensation.

Index Terms—compressed sensing, dynamic MRI

1. INTRODUCTION

Recent theoretical and practical advances in compressed sensing (CS) of images and video has spawned interest in applying CS methodology to sequences of magnetic resonance imagery (MRI). Such temporally varying imagery is commonly referred to as dynamic MRI, and several algorithms have been devised specifically for its reconstruction from CS measurements, e.g., [1–5]. While CS-based sampling has the potential to greatly accelerate the acquisition process of dynamic-MRI data, by capitalizing on methodologies such as motion estimation (ME) and motion compensation (MC) that arise in conventional video processing, we can exploit temporal redundancy existing in dynamic MRI to improve CS reconstruction. Specifically, in this paper, we propose a CS reconstruction for dynamic MRI that is inspired by an ME/MC-based CS framework that we developed previously for conventional video [6–8]. That is, given a collection of subsampled k-space (frequency-space) measurements, we reconstruct a projected space (frequency-space) measurement of the frame in the space of the frequency measurements. That is, given a collection of subsampled k-space measurements, we reconstruct a projected space (frequency-space) measurement of the frame in the space of the frequency measurements.

2. BACKGROUND

The CS paradigm concerns the reconstruction of a sparse (or compressible) signal \( x \) with length \( N \) from \( M \) measurements \( y = \Phi x \) where \( M \ll N \), \( \Phi \) is an \( M \times N \) measurement matrix, and \( S = M/N \) is subsampling ratio or subrate. For MRI data, a partial Fourier matrix is commonly used for \( \Phi \) (e.g., Fig. 8 in [11]).

One effective method for CS reconstruction of \( x \) from measurements \( y \) is a PL algorithm [10, 12–14], which has also been called “iterative thresholding” (e.g., [15, 16]). PL starts from some initial approximation \( \tilde{x}^{(0)} \) and forms the approximation at iteration \( i + 1 \) as

\[
\tilde{x}^{(i+1)} = \begin{cases} 
\tilde{x}^{(i)} + \frac{1}{\gamma} \Psi \Phi^T \left( y - \Phi \Psi^{-1} \tilde{x}^{(i)} \right), & \text{if } \tilde{x}^{(i)} \geq \tau^{(i)} \\
0, & \text{otherwise}
\end{cases}
\]

(1)

Here, \( \gamma \) is a scaling factor ([13] uses the largest eigenvalue of \( \Phi^T \Psi \)), while \( \tau^{(i)} \) is a threshold set appropriately at each iteration. Advantages of PL-based CS reconstruction include reduced computational complexity along with flexibility of choosing a sparsity transform \( \Psi \) as well as the thresholding method.
3. MOTION-COMPENSATED CS FOR DYNAMIC MRI
3.1. PL with Directional Transforms

The PL algorithm based on (1) and (2) and designed for MRI is as follows:

\[
\begin{align*}
\text{function } x^{(i+1)} &= \text{PL}(x^{(i)}, y, \Phi, y^i, y, \lambda) \\
\bar{x}^{(i)} &= \bar{x}^{(i)} + \lambda_t \cdot (\hat{x}^{(i)} - \bar{x}^{(i)}) \\
\hat{x}^{(i)} &= \text{Threshold}(\bar{x}^{(i)}, \lambda) \\
\bar{x}^{(i)} &= \bar{x}^{(i)} + \lambda_t \cdot (\hat{x}^{(i)} - \bar{x}^{(i)}) \\
\end{align*}
\]

Here, \(\text{Threshold}(\cdot)\) is a thresholding process as discussed below.

In our use of PL, we initialize with \(x^{(0)} = \Phi^T y\) and terminate when \(|D^{(i+1)} - D^{(i)}| < 10^{-4}\), where \(D^{(i)} = \frac{1}{\sqrt{N}} \|x^{(i)} - x^{(i-1)}\|_2\). For \(\Phi\), we use a dual-tree discrete wavelet transform (DDWT) [17], the effectiveness for CS reconstruction of which was demonstrated in [10]. For thresholding, we apply bivariate shrinkage [18] on the DDWT coefficients as was done in [10]. Specifically, for each coefficient of \(\hat{x}^{(i)}\),

\[
\text{Threshold}(\hat{x}, \lambda) = \frac{(\sqrt{\hat{x}^2 + \lambda_t^2} - \lambda_t \sigma_s^{(i)} \cdot \hat{x})_+}{\sqrt{\hat{x}^2 + \lambda_t^2}}
\]

where \((g)_+ = 0\) for \(g < 0\), \((g)_+ = g\) else; \(\hat{x}_r\) is the coefficient in the parent decomposition level; \(\sigma_s^{(i)}\) and \(\sigma_s\) are noise and signal variance, respectively; and \(\lambda\) is a convergence-control factor.

We refer to the overall process—which will form the foundation of the dynamic-MRI reconstruction discussed next—as CS-PL. Example reconstruction of CS-PL is compared to well-known TV reconstruction (e.g., [19]) in Fig. 1 for a single (2D) MRI image. While the visual quality is similar, CS-PL is around 20 times faster than TV. We consequently use CS-PL as the base reconstruction in the sequel.

3.2. Residual Reconstruction

Based on the assumption that MR images have high temporal correlation, one frame can be enhanced by the information from the previously reconstructed frames. Suppose we have measurements, \(y\), of the current frame, \(x\), and its prediction \(y_{\text{pred}}\) where the latter is obtained by a ME/MC process using previously reconstructed frame(s). Instead of a straightforward 2D reconstruction using \(y\), we can reconstruct the projected residual of the measurements to exploit temporal correlation; i.e.,

\[
y_t = y - \Phi \hat{x}_{\text{pred}}.
\]

It is clear that \(y_t\) is the projection of the residual, \(y\), between our prediction \(\hat{x}_{\text{pred}}\) and the original and still-unknown \(x\); i.e.,

\[
y_t = y - \Phi \hat{x}_{\text{pred}} = \Phi (x - \hat{x}_{\text{pred}}) = \Phi x_c.
\]

If the prediction process is accurate, the residual frame \(x_c\) should be more compressible than the original frame \(x\), so its reconstruction should be more accurate. Consequently, we can form a new approximation to \(x\) as

\[
x = \hat{x}_{\text{pred}} + \hat{x}_r.
\]

This process is called residual reconstruction (e.g., [2, 6–9]). We now have a new approximation to the current frame that is of better quality than the initial approximation that we created from a direct CS-PL reconstruction from \(y\). We turn our attention to the issue of producing the prediction of the current frame next.

3.3. ME/MC with Telescopic Search

In traditional video processing, ME/MC is used to reduce temporal redundancy by tracking object motion from frame to frame. ME is applied between two frames to estimate a motion field, and such ME could be performed in a variety of ways. For simplicity, we consider full-search, block-based ME. Consequently, the initial reconstruction of the current frame is partitioned into blocks whose motion from a reference frame is determined by a block-matching search. We note that we group a number of consecutive frames together as a group of pictures (GOP) as is commonly done in traditional video processing, and the motion search is performed within the GOP. The first frame of the GOP is denoted as a “key frame” which typically has a higher subrate than the other “non-key frames” of the GOP.

We have previously employed ME/MC for the reconstruction of conventional video from CS measurements (e.g., [6–8]). In conventional video, block-based ME/MC performs well when objects...
undergo geometric change (i.e., translation) from one frame to the next. In case of certain dynamic-MRI sequences, a motion search using only the immediately preceding frame is insufficient to find a good match which might exist in the sequence long before. An example of such a sequence is the dynamic MRI of a heart, wherein contractions and expansions of a cardiac chamber repeat at a certain interval; in this case, the best match for the current block might lie a number of frames previous, depending on the frequency of the cardiac pulsations.

More effective block matching in sequences with repetitive patterns can be obtained by searching over several previous frames using a telescopic search [21–23], illustrated in Fig. 2 for dynamic MRI. The previous frame is used as the reference for the usual block search for the current block in the current frame. The location of the resulting best-matching block in the reference frame is then in turn used as the start location for a motion search in the immediately preceding frame, and this process is repeated until the end of the GOP is reached. Then, the best match from any of the searched frames becomes the final prediction.

We extend the CS-PL of the previous section by incorporating telescopic ME/MC, resulting in the MC-CS-PL algorithm that reconstructs the current frame at time $t$:

$$\hat{x}_t = \text{MC-CS-PL}(y_t, \Phi, \Psi, \hat{x}_{t-1}, \hat{X}, \omega)$$

Here, $\text{MotionCompensation}(\cdot)$ implements block-based single-reference-frame ME/MC, $\text{MotionCompensationTelescopic}(\cdot)$ is the telescopic-search ME/MC applied to the entire GOP, $y_t$ is the set of measurements for the current frame, $\hat{x}_{t-1}$ is the previously reconstructed preceding frame, $\hat{X}$ is the set of $P$ previously reconstructed frames for the $P$-frame GOP that contains the current frame, and $\omega$ is a weighting factor. We note that the reconstruction of the current frame, $\hat{x}_t$, produced by MC-CS-PL is placed into the GOP $\hat{X}$, replacing the corresponding frame there, prior to the application of MC-CS-PL to the next frame.

As for weight $\omega$, we have found that restricting ME/MC to simply the preceding frame works well at low subrate; however, as the subrate increases, the telescopic search tends to work better. As a consequence, we balance the two forms of ME/MC with weight $\omega$ ($0 \leq \omega \leq 1$) depending on the subrate of the non-key frames; i.e.,

$$\omega = \begin{cases} 0.2, & S_{NK} < 0.2, \\ 0.8, & S_{NK} > 0.5, \\ 2S_{NK} - 0.2, & \text{otherwise}, \end{cases}$$

(7)

where $S_{NK}$ is subrate for non-key frame$^2$.

To obtain the initial reconstructions for the GOP, $\hat{X}$, as needed to start MC-CS-PL, we apply the following procedure to produce each $\hat{x}_t$ of the GOP:

$$\hat{x}_{\text{init}} = \text{Initialize}(y, \Phi, \Psi, \hat{x}_{t-1}, \omega)$$

$$\hat{x}' = \text{CS-PL}(y, \Phi, \Psi)$$

$^2$We fix $S_K$, the subrate for the key frame that starts the GOP, to be 0.7.

An initial reconstruction can be produced via 2D CS-PL or simple use of the previously reconstructed frame as a (non-ME/MC) prediction. Empirical results in [7] show that a weighted combination of both approaches outperforms either approach used alone. Here, $\omega$ is the same weight from (7).

Finally, we note that we have described the MC-CS-PL here in terms of forward motion prediction [23] wherein the current frame is predicted by a preceding frame. However, as described in [6, 7], we can equally use backward prediction in which the current frame is predicted by a subsequent frame. Consequently, we implement MC-CS-PL using forward prediction for the first half of the GOP, backward prediction for the last half of the GOP, and both for the center frame.
4. EXPERIMENTAL RESULTS

We now examine the performance of MC-CS-PL reconstruction relative to a corresponding “intraframe” reconstruction to demonstrate that significant gain results from the explicit exploitation of motion information within the CS reconstruction of dynamic MRI. We also compare to two prominent CS reconstruction algorithms, Modified-CS-Residual [3] and k-t FOCUSS with ME/MC [1, 2]. We use implementations of k-t FOCUSS\(^3\) and Modified-CS-Residual\(^4\) available from their respective authors.

We use the MRI sequences “Cardiac” (24 frames) and “Larynx” (56 frames). Both of sequences are grayscale frames with spatial size of \(256 \times 256\). The sequences are subject to partial Fourier projection applied frame by frame; for MC-CS-PL, we use a radial sensing trajectory with uniformly spaced rays (as in \(l_1\)-MAGIC\(^5\)) for measurement operator \(\Phi\). For other techniques, we use the default measurement process used in the respective software implementations. In all cases, we use a GOP size of \(P = 8\) frames with key frames starting each GOP having a subrate of \(S_K = 0.7\). The intervening non-key frames have subrate \(S_{NK}\) varying between 0.1 and 0.5. As a primary measure of reconstruction quality, we calculate the PSNR averaged over all non-key frames under consideration. For the ME/MC process in MC-CS-PL, we use full-search ME with full-pixel accuracy, a block size of 32, a search window of \(\pm 7\) pixels, and a telescopic search range within the respective GOP. We note that k-t FOCUSS uses an overlapped search, a block size of 2, and a search window of \(\pm 7\) by default in the original software package.

First, we compare MC-CS-PL with simpler variants of the same, namely “intraframe” reconstruction (i.e., CS-PL applied independently frame by frame) and MC-CS-PL without telescopic search. We see in Fig. 3 that both of the MC-CS-PL techniques achieve higher-fidelity reconstruction especially at low subrate, meaning that exploiting motion information enhances intraframe reconstruction. Additionally, the results show that the telescopic search results in improved performance at higher subrates.

Next, we compare MC-CS-PL to k-t FOCUSS as well as Modified-CS-Residual in Figs. 4 and 5. These results indicate that MC-CS-PL with telescopic search outperforms the other techniques considered, sometimes by as much as 5–8 db. Finally, Fig. 6 illustrates sample visual results for the “Cardiac” sequence. Because Modified-CS-Residual inherently keeps only the most significant transform coefficients over the frames, it misses some detail DWT coefficients and thereby does not capture the contours of the organs well. On the other hand, k-t FOCUSS yields clear and sharp contours albeit with some blurring and smearing elsewhere. On the other hand, MC-CS-PL result appears to capture most detail of the original frame without significant distortion.

5. CONCLUSIONS

In this paper, we examined the use of ME/MC in CS recovery of dynamic MRI. For 2D reconstruction, a directional transform and statistical thresholding are plugged into PL to capture the sparse pattern in the MRI while keeping directional features of the image. For ME/MC, we find that a spatially narrow search over a long temporal range results in better block matches in order to capture cyclic motion pattern due to the repetitive behavior of internal organ. Incorporating a telescopic motion search into CS-PL reconstruction, the resulting MC-CS-PL algorithm outperforms both k-t FOCUSS and Modified-CS-Residual, prior CS reconstruction techniques designed specifically for dynamic MRI.
REFERENCES


