Security-Constrained Unit Commitment With AC Constraints*

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Abstract—In a restructured power market, the independent system operator (ISO) executes the security-constrained unit commitment (SCUC) program to plan a secure and economical hourly generation schedule for the day-ahead market. This paper introduces an efficient SCUC approach with ac constraints that obtains the minimum system operating cost while maintaining the security of power systems. The proposed approach applies the Benders decomposition for separating the unit commitment (UC) in the master problem from the network security check in subproblems. The master problem applies the augmented Lagrangian relaxation (LR) method and dynamic programming (DP) to solve UC. The subproblem checks ac network security constraints for the UC solution to determine whether a converged and secure ac power flow can be obtained. If any network violations arise, corresponding Benders cuts will be formed and added to the master problem for solving the next iteration of UC. The iterative process will continue until ac violations are eliminated and a converged optimal solution is found. In this paper, a six-bus system and the IEEE 118-bus system with 54 units are analyzed to exhibit the effectiveness of the proposed approach.

Index Terms—Augmented Lagrangian relaxation, Benders decomposition, network security constraints, restructured power markets, security-constrained unit commitment (SCUC).

NOMENCLATURE

- $b$ Index for bus.
- $c$ Coefficient in the augmented Lagrangian function.
- $C_{b1}, C_{b2}$ Slack variables for bus voltage constraints at bus $b (\geq 0)$.
- $C_{t1}, C_{t2}$ Slack variables for transmission flow constraints on line $l (\geq 0)$.
- $d_l$ Penalty cost of transmission flow violation on line $l$.
- $DR_i$ Ramp-down rate limit of unit $i$.
- $e_b$ Penalty cost of voltage violation on bus $b$.
- $E_{S}^{\text{max}}$ System emission limit.
- $F_{C}(\cdot)$ Production cost function of unit $i$.
- $F_{R}(\cdot)$ Fuel consumption function of unit $i$.
- $F_{\text{em}}(\cdot)$ Emission function of unit $i$.
- $F_{\text{full}}(\cdot)$ Minimum fuel consumption (type FT).
- $F_{\text{max}}(\cdot)$ Maximum fuel consumption (type FT).
- $FT$ Index for fuel type.
- $I_{it}$ Index for iteration in the master problem.
- $k$ Index for line.
- $l$ Slack variables for real power mismatch at bus $l (\geq 0)$.
- $M_{Q_{b1}, M_{Q_{b2}}}$ Slack variables for reactive power mismatch at bus $b (\geq 0)$.
- $n$ Number of iterations between master problem and subproblem.
- $N_{L}$ Number of lines.
- $N_{N}$ Number of units.
- $N_{B}$ Number of buses.
- $N_{T}$ Number of periods under study (24 h).
- $P_{D,t}$ System demand at time $t$.
- $P_{L,t}$ System losses at time $t$.
- $P_{it}$ Generation of unit $i$ at time $t$.
- $P_{i}^{\text{amin}}$ Lower limit of real power generation of unit $i$.
- $P_{i}^{\text{amin}}$ Upper limit of real power generation of unit $i$.
- $P_{i}^{\text{max}}$ Initial real power flow on line $l$.
- $RL_{i}$ Maximum capacity of line $l$.
- $Q_{t}^{\text{amin}}$ Real power on line $l$ and its increment.
- $Q_{i}^{\text{max}}$ Lower limit of reactive power generation of unit $i$.
- $Q_{i}^{\text{max}}$ Upper limit of reactive power generation of unit $i$.
- $R_{S,t}$ System spinning reserve requirement at time $t$.
- $R_{O,t}$ System operating reserve requirement at time $t$.
- $R_{S,it}$ Spinning reserve of unit $i$ at time $t$.
- $R_{O,it}$ Operating reserve of unit $i$ at time $t$.
- $SD_{C,i}$ Shutdown fuel consumption of unit $i$ at time $t$.
- $SD_{D,i}$ Shutdown emission of unit $i$ at time $t$.
- $SD_{C,i}$ Shutdown cost of unit $i$ at time $t$.
- $SU_{S,i}$ Startup fuel consumption of unit $i$ at time $t$.
- $SU_{S,i}$ Startup emission of unit $i$ at time $t$.
- $SU_{N}$ Startup cost of unit $i$ at time $t$.
- $T_{t}$ Index for time.
- $T_{d}$ Minimum down time of unit $i$.
- $T_{u}$ Minimum up time of unit $i$.
- $UR_{i}$ Ramp-up rate limit of unit $i$.
- $V_{b0}$ Initial voltage magnitude at bus $b$.
- $V_{b0}^{\text{amin}}$ Minimum voltage magnitude at bus $b$.
- $V_{b0}^{\text{max}}$ Maximum voltage magnitude at bus $b$.


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\( V_b, \Delta V_b \)  
Voltage magnitude at bus \( b \) and its increment.

\( X_{\text{off}} \)  
OFF time of unit \( i \) at time \( t \).

\( X_{\text{on}} \)  
ON time of unit \( i \) at time \( t \).

\( X_{\text{km}} \)  
Reactance of line \( k \)-\( m \).

\( Y \)  
Bus-unit incidence matrix.

\( \lambda_t \)  
Lagrangian multiplier for power balance constraint at time \( t \).

\( \mu_{\text{Sp},t} \)  
Lagrangian multiplier for spinning reserve constraint at time \( t \).

\( \mu_{\text{Or},t} \)  
Lagrangian multiplier for operating reserve constraint at time \( t \).

\( \eta_{\text{Ft}} \)  
Lagrangian multiplier for fuel limit of fuel type FT.

\( K \)  
Lagrangian multiplier for system emission limit.

\( \nu \)  
Lagrangian multiplier for Benders cut.

\( \pi_{\text{In}} \)  
Marginal change in violations with a 1-MW increase in the unit \( i \) generation at time \( t \) associated with the \( n \)th iteration.

\( \Phi \)  
Denoting known variables.

\( B' \)  
Susceptance matrix.

\( I \)  
Unit state vector.

\( dP_0 \)  
Initial real power mismatch vector.

\( dQ_0 \)  
Initial reactive power mismatch vector.

\( P_r, P_g \)  
Unit real power generation vector.

\( P_d \)  
Real power nodal load vector.

\( \Delta P \)  
Generating unit real power increment vector.

\( \Delta Q \)  
Generating unit reactive power increment vector.

\( \delta, \Delta \delta \)  
Bus phase angle vector and its increment vector.

\( \Delta V \)  
Bus voltage increment vector.

\( T, \Delta T \)  
Transformer tap vector and its increment vector.

\( \gamma, \Delta \gamma \)  
Phase shifter angle vector and its increment vector.

\( T_0 \)  
Initial transformer tap vector.

\( \gamma_0 \)  
Initial phase shifter angle vector.

\( \Delta Q_{\text{min}}, \Delta Q_{\text{max}} \)  
Lower and upper limit vector of generating unit reactive power increment.

\( T_{\text{min}}, T_{\text{max}} \)  
Lower and upper limit vector of transformer tap.

\( \gamma_{\text{min}}, \gamma_{\text{max}} \)  
Lower and upper limit vector of phase shifter angle.

\( \pi, \psi, \phi \)  
Simplex multiplier vector.

\( H, N, E, F, M, J \)  
Jacobian matrices.

\( R, S, A, B, C, D \)  
More Jacobian matrices.

I. INTRODUCTION

The generation business is rapidly becoming market driven. However, the system security is still the most important aspect of the power system operation and cannot be compromised in a market-driven approach. Market operators in various ISOs apply the standard market design (SMD) for scheduling a secure and economically viable power generation for the day-ahead market. One of the key components of SMD is security-constrained unit commitment (SCUC), which utilizes the detailed market information submitted by participants, such as the characteristics of generating units, availability of transmission capacity, generation offers and demand bids, scheduled transactions, curtailment contracts, and so on. SCUC provides a financially viable unit commitment (UC) that is physically feasible. The generation dispatch based on SCUC is made available to corresponding market participants. The market participants could use the available signals for reconsidering their proposed bids on generating resources, which includes signals on LMPs and transmission congestion. Normally, an acceptable SCUC solution could be reached in cooperation with market participants if the day-ahead market is healthy and robust.

Fig. 1 depicts the hierarchy for calculating SCUC, which is based on the existing setup in restructured power systems. The hierarchy utilizes a Benders decomposition, which decouples the SCUC into a master problem (UC) and a network security check subproblem.

The initial UC and economic dispatch (ED) of generating units are obtained in the optimal generation block based on the available market information. Then, the ac network security block checks the constraints and tries to minimize any network security violations. However, if violations persist, certain constraints (Benders cuts) will be passed along to the optimal generation block for recalculating the UC solution. The iterative process will continue until all violations are eliminated and a converged optimal solution is found.

In order to satisfy the network security constraints, replaced transmission constraints with penalty functions that appear directly in the Lagrangian function. In other words, all transmission constraints would be relaxed by using multipliers that are included in UC. The addition of multipliers could make it more difficult and even impossible to obtain the optimal UC solution as the number of constraints becomes larger.

We presented a transmission-constrained unit commitment in [12], which utilized Benders decomposition. However, the study did not consider voltage limits as part of generation scheduling. The lack of sufficient reactive power generation could result in higher losses and perhaps the infeasibility of ac power flow solution. We presented in [13] a UC with transmission security and voltage constraints, which employed two separate subproblems.
for checking transmission and voltage constraints. The real and reactive power constraints were handled by two separate subproblems, which could ignore the impact of real power adjustment on reactive power constraints and vice versa. Accordingly, the convergence of ac power flow could not be guaranteed based on UC and ED solutions.

In this paper, constraints on reactive power are not taken into account in the initial master problem (UC). However, the corresponding adjustments of VAR resources are included in the hourly network security check subproblems. If the ac power flow cannot converge or any violations of transmission flows and bus voltages exist, the corresponding Benders cut will be formed and fed back into the next calculation of the master problem (UC). This cut represents the coupled information on real and reactive power adjustments. In essence, Benders coefficients represent the interactions between real and reactive power constraints.

In comparison with our earlier SCUC studies, a full Newton–Raphson method is applied in this paper for minimizing system security violations as we learned that the decoupled load flow model might pose shortcomings in the ac modeling of network constraints. More stringent generation and network constraints are imposed to reflect the SMD in restructured power systems. Furthermore, we learned that in order to get a better initial state and improve the performance of SCUC with ac constraints, we should initially execute the SCUC with dc constraints. This approach is implemented in our proposed SCUC algorithm.

The rest of the paper is organized as follows. Section II provides the mathematical model of SCUC with the prevailing constraints and discusses the solution methodology. Section III presents and in detail discusses a six-bus system with three units and the IEEE 118-bus system with 54 units. The conclusion drawn from the study is provided in Section IV.

II. SCUC PROBLEM FORMULATION AND METHODOLOGY

A. SCUC Formulation

As discussed in the Introduction, the objective of SCUC is to determine a day-ahead UC for minimizing the system operating cost while meeting the prevailing constraints listed as follows:

1) Power balance
2) Hourly generation bids
3) Must-on and area protection constraints
4) System spinning and operating reserve requirements
5) Minimum up and down time limits
6) Ramp rate limits
7) Startup and shutdown characteristics of units
8) Fuel and multiple emission constraints
9) Transmission flow and bus voltage limits
10) Load shedding and bilateral contracts
11) Limits on state and control variables including real and reactive power generation, controlled voltages, and settings of tap-changing and phase-shifting transformers
12) Scheduled outages

The list of symbols is presented in the Nomenclature section. We consider a thermal unit commitment in which the objective is to minimize the cost of supplying the load as formulated below:

$$\min \sum_{i=1}^{NG} \sum_{t=1}^{NT} [F_{ci}(P_{it}) \ast I_{it} + SU_{it} + SD_{it}].$$

Function (1) is composed of fuel costs for producing electric power and startup and shutdown costs of individual units over the given period.

Generation constraints listed next include the system power balance (2), system spinning and operating reserve requirements (3), ramping up/down limits (4), minimum up/down time limits (5), and real and reactive power generation limits (6), (7). Additional system-wide constraints such as fuel constraints (8) and emission limits (9) are included in this formulation for representing the interactions among electricity market, fuel market, and environment.

$$\sum_{i=1}^{NG} P_{it} \ast I_{it} = P_{D,t} + P_{L,t} \quad (t = 1, \ldots, NT)$$

$$\sum_{i=1}^{NG} R_{Si,t} \ast I_{it} \geq R_{S,t} \quad (t = 1, \ldots, NT)$$

$$\sum_{i=1}^{NG} R_{Oi,t} \ast I_{it} \geq R_{O,t} \quad (t = 1, \ldots, NT)$$

$$P_{it} - P_{it(t-1)} \leq [1 - I_{it} \left(1 - I_{it(t-1)}\right)] UR_i + I_{it} \left(1 - I_{it(t-1)}\right) \cdot P_{t,\text{min}}$$

$$P_{t(t-1)} - P_{it} \leq [1 - I_{it} \left(1 - I_{it(t-1)}\right)] DR_i + I_{it} \left(1 - I_{it(t-1)}\right) \cdot P_{t,\text{min}}$$

$$\left[X_{\text{on}}^{i(t-1)} - T_{\text{on}}^{i}\right] \ast [I_{it} - I_{it(t-1)}] \geq 0$$

$$\left[X_{\text{off}}^{i(t-1)} - T_{\text{off}}^{i}\right] \ast [I_{it} - I_{it(t-1)}] \geq 0$$

$$P_{i,\text{min}} \cdot I_{it} \leq P_{it} \leq P_{i,\text{max}} \cdot I_{it} \quad (i = 1, \ldots, NG)(t = 1, \ldots, NT)$$

$$Q_{i,\text{min}} \cdot I_{it} \leq Q_{it} \leq Q_{i,\text{max}} \cdot I_{it} \quad (i = 1, \ldots, NG)(t = 1, \ldots, NT)$$

$$F_{\text{F,T}}^{\text{min}} \leq \sum_{T=1}^{NT} \sum_{i \in FT} [F_{ci}(P_{it}) \ast I_{it} + SU_{fi,t} + SD_{fi,t}] \leq F_{\text{F,T}}^{\text{max}}$$

$$\sum_{T=1}^{NT} \sum_{i \in FT} [F_{ci}(P_{it}) \ast I_{it} + SU_{ei,t} + SD_{ei,t}] \leq E_{S}^{\text{max}}.$$
state and contingency cases. In this paper, we exclude ac network contingencies in SCUC, which will be discussed further in a subsequent article

\[ -\text{PL}_{d,\text{MAX}}^t \leq \text{PL}_d^t \leq \text{PL}_{d,\text{MIN}}^t \quad (t = 1, \ldots, NL) \quad (t = 1, \ldots, NT) \]  

(10)

\[ V_{b,\text{MIN}}^t \leq V_b^t \leq V_{b,\text{MAX}}^t \quad (b = 1, \ldots, NB) \quad (t = 1, \ldots, NT). \]  

(11)

Constraints (12) and (13) are limits on tap changing and phase shifting transformer settings

\[ T_{\text{MIN}}^t \leq T^t \leq T_{\text{MAX}}^t \quad (t = 1, \ldots, NT) \]  

(12)

\[ \gamma_{\text{MIN}}^t \leq \gamma^t \leq \gamma_{\text{MAX}}^t \quad (t = 1, \ldots, NT). \]  

(13)

B. Decomposition Strategy for SCUC

Benders decomposition is a popular optimization technique. In applying the Benders decomposition algorithm, the original large-scale optimization problem will be decomposed into a master problem and several subproblems, based on the linear programming (LP) duality theory. Generally, the master problem is an integer program, and subproblems are linear programs that deal with real variables.

In order to employ the decomposition strategy for SCUC, we assume that \( x \) represents the UC state \( I \) and dispatch \( P \) and that \( y \) represents the system state and control variables \( Q, \delta, V, T, \) and \( \gamma \). Once the cost function (1) and constraints (8)–(10) are linearized [1], the SCUC problem is rewritten as in the following standard Benders formulation:

\[
\begin{align*}
\text{Min} & \quad c^T x \\
\text{S.t.} & \quad Ax \geq b \\
& \quad Ex + Fy \geq h
\end{align*}\]  

(14–16)

where (15) represents UC constraints (2)–(6) and (8)–(9), and (16) represents the remaining constraints, including network security constraints and limits on system state and control variables.

Accordingly, the initial master problem is as follows:

\[
\begin{align*}
\text{Min} & \quad c^T x \\
\text{S.t.} & \quad Ax \geq b.
\end{align*}\]  

(17)

Then, in order to check whether (16) is satisfied based on \( \hat{x} \) given in the master problem, a slack vector \( s \) is introduced and the corresponding subproblem is formulated as

\[
\begin{align*}
\text{Min} & \quad w(x) = 1^T s \\
\text{S.t.} & \quad Fy + s \geq h - Ex \quad \pi.
\end{align*}\]  

(18)

Here, \( 1 \) is the vector of ones, and \( w(\hat{x}) > 0 \) means that violations occur in the subproblem. \( \pi \) is the simplex multiplier vector of inequality constraints in (18). \( \pi^T E \) mathematically represents the marginal increment/decrement of the objective value when \( x \) is changed. In order to eliminate the violations, the Benders cut (19) is introduced and added to the master problem

\[
w(x) = w(\hat{x}) - \pi^T E(x - \hat{x}) \leq 0. \]  

(19)

The final solution based on the Benders decomposition algorithm may require an iterative process between the master problem and the subproblem.

C. Master Problem of SCUC

The initial master problem of SCUC provides a commitment and dispatch for minimizing the operating cost of available generators while disregarding the network security constraints. The UC problem has an objective function (1) and constraints (2)–(6) and (8)–(9), as we consider (7) in the subproblem.

Mathematically, UC is a nonconvex, nonlinear, large-scale, mixed-integer optimization problem with a great number of 0-1 scheduling variables, continuous and discrete control variables, and a series of prevailing equality and inequality constraints [14]. Various techniques, such as DP, LR, mixed-integer programming (MIP), genetic algorithms, and expert systems, have been studied to achieve a near-optimal feasible solution for minimizing the operating cost while satisfying all or partial generation constraints. However, bottlenecks, including DP’s high dimensionality, LR’s limitations on minimizing the duality gap, MIP’s slow and even infeasible convergence due to the many 0-1 variables, and the fine tuning of solution based on genetic algorithms and expert systems, yield a barrier for practical applications [15]–[17].

In LR, unless a proper modification of multipliers is ensured in every iteration, unnecessary commitment of generating units may occur, which may result in higher operating costs. These difficulties are often explained by the nonconvexity of this type of optimization problems. In this paper, a DP approach based on augmented LR is applied to solve the UC problem at a reasonable calculation speed. To implement augmented LR, we add a quadratic penalty term \( (c/2) \sum_{t=1}^{NT} (\sum_{i=1}^{NG} P_{d}^i * I_{d}^i - P_{D}^t)^2 \) to the Lagrangian function for improving the convexity of the problem and the convergence of the algorithm [3]. Accordingly, the nonseparable quadratic penalty terms are linearized around the solution of the previous iteration. The decoupled augmented Lagrangian function for unit \( i \) at the \( (k+1) \)th iteration is given in (20)

\[
L = \sum_{t=1}^{NT} [F_{E}(P_{d}^t) * I_{d}^t + SU \sum_{at=1}^{NT} \hat{\lambda}_{b}^t * P_{d}^t * I_{d}^t \]  

\[- \sum_{t=1}^{NT} \hat{\mu}_{b}^t * R_{b}^t * I_{d}^t - \sum_{t=1}^{NT} \hat{\rho}_{S}^t * R_{S}^t * I_{d}^t \]  

\[+ \hat{\eta}_{F}^T \sum_{t=1}^{NT} [F_{E}(P_{d}^t) * I_{d}^t + SU \sum_{f=1}^{NT} D_{f}^t * I_{d}^t + SD_{f}^t * I_{d}^t] \]  

\[+ \hat{k}_{S}^T \sum_{t=1}^{NT} [F_{E}(P_{d}^t) * I_{d}^t + SU \sum_{c=1}^{NT} D_{c}^t * I_{d}^t + SD_{c}^t * I_{d}^t] \]  

\[+ \sum_{t=1}^{NT} c_{t} * P_{d}^t * I_{d}^t * \left( P_{D}^t - \sum_{i=1}^{NG} (P_{d}^i * I_{d}^i) \right) \]  

\[+ \sum_{t=1}^{NT} c_{t} * P_{D}^t * P_{d}^t * I_{d}^t \]  

\[- \frac{1}{2} \sum_{t=1}^{NT} (P_{d}^t * I_{d}^t - P_{d}^t * I_{d}^t)^2 \]  

\[+ \sum_{t=1}^{NT} \hat{\mu}_{b}^t * \pi_{b}^t * P_{d}^t * I_{d}^t. \]  

(20)
In this formulation, constraints (2), (3), (8), and (9) are relaxed by applying Lagrangian multipliers $\lambda, \mu, \eta,$ and $\kappa$. In addition, the exactly convex quadratic terms of decision variables are added as auxiliary functions to improve the convergence property, which are strongly separable, convex, and differentiable with respect to $P^{a}_u$. Note that constant terms are omitted and that the last term in (20) represents relaxed network violation cuts after the network violations are determined. $P^{a}_u$ and $P^{b}_u$ are obtained from the previous iteration. $\varepsilon$ is a positive number and $1/\varepsilon \geq 2c$. The value of $c$ is computed as the difference between the largest and the smallest system lambdas divided by the difference between load values in the respective intervals [4].

Fig. 2 depicts the flowchart of UC. The procedure devised in this figure is as follows.

Step 1) The initial Lagrangian multipliers are set, which include $\lambda$ for power balance equalities, $\mu$ for reserve requirements, $\eta$ for system fuel limits, $\kappa$ for system emission limits, and $\nu$ for system security constraints (Benders cuts).

Step 2) The relaxed problem is decoupled into subproblems representing individual units (20). DP is used, based on the current values of multipliers, to solve the UC for each unit over a 24-h period.

Step 3) The power balance, reserve, fuel, and emission constraints as well as any Benders cuts from the network security check subproblem are checked. The multipliers are updated by applying the subgradient method [14], and Step 2 is repeated, if constraints cannot be satisfied. Otherwise, the algorithm will execute Step 4.

Step 4) The dual objective $q^*$ in the Lagrangian function and the primal objective function $J^*$ (i.e., ED over a 24-h period) are calculated. The algorithm will be terminated in the master problem if the relative duality gap is within the tolerance $\varepsilon$. Otherwise, multipliers will be updated via the subgradient method, and the algorithm will return to Step 2.

D. Subproblem of SCUC

Once the initial UC and ED are calculated by the master problem, the network security check subproblem is solved for 24 h, as shown in Fig. 3. We use two subproblems in Fig. 3 to check network security constraints. The first one guarantees the convergence of power flow, and the second one guarantees the transmission network security.

At first, real and reactive power mismatches at various buses are considered for minimization to check whether a converged ac power flow solution can be obtained based on UC results. A mismatch power within its limit will guarantee a converged power flow solution. If the total mismatch exceeds the specified limit, a mismatch violation cut will be generated for the SCUC master problem.

Next, transmission flow and bus voltage violations will be checked. The adjustments in the subproblem are based on committed power generation, phase shifters, tap-changing transformers, and other control devices. If any violations persist, the subproblem will form the corresponding Benders cuts for the master problem in the next iteration of UC. The mitigation of violations will result in the final SCUC solution.

If violations are not mitigated after a certain number of iterations between the master problem and the subproblem, load shedding will be prescribed based on the submitted decremental load bids. Load shedding in Fig. 3 is modeled by adding virtual generators at load buses with permissible load shedding contracts. Network security violations will be eliminated once the virtual generators curtail a certain amount of load at local buses.

In this paper, the Newton–Raphson method is used for solving the network flow equations because of its quadratic convergence. This method is less prone to divergence in ill-conditioned problems and gives a complete and efficient consideration to interactions between real and reactive power.
transmission flows and phase shifters, and bus voltages and tap-changing transformers, while demonstrating a satisfactory convergence speed. LP is used for the solution of network security check subproblem and load shedding. The use of LP provides a great flexibility in the modeling of subproblem and allows an easy implementation of new features [17], [18].

In order to get a better initial state and improve the performance of SCUC with ac constraints, we initially execute the SCUC with dc constraints. The dc model, which is much easier to analyze, is formulated as follows.

1) DC Network Security Subproblem: In this section, we present a method for correcting transmission overflows based on dc power flow equations. In order to check transmission flows and gain a feasible solution in the case of flow violations, we add slack variables to transmission constraints. Slack variables are interpreted as the amount of transmission constraint violations associated with the UC state $I(t)$ and the related dispatch $P_g$ at hour $t$. The hourly network security check subproblem with dc constraints is formulated as

$$\begin{align*}
\text{Min } & \quad w(I, P_g) = \sum_{t=1}^{NL} (C_{t,1} + C_{t,2}) \\
\text{s.t.} & \quad (P_{L,t} - C_{t,1}) \leq P_{L,t,\text{max}} \\
& \quad - (P_{L,t} + C_{t,2}) \leq P_{L,t,\text{max}} \\
& \quad - \infty \leq P_{L,t} \leq +\infty \\
& \quad P = \bar{P}_g \bar{I} \\
& \quad \delta_{\text{ref}} = 0 \\
& \quad \gamma_{\text{in}t} \leq \gamma \leq \gamma_{\text{max}} \\
& \quad B^* \delta = Y^* P - P_d \\
& \quad PL = \frac{\delta_k - \delta_m - \gamma_{\text{in}t}}{\lambda_{km}} \quad (k, m \in I).
\end{align*}$$

In this subproblem, transmission flow violations are minimized by adjusting phase shifter angles (control variable $\gamma_{km}$). When $w(I, P_g) > 0$, Benders cuts (30) are generated and added to the master problem. Benders cuts indicate that current transmission flow violations ($\dot{w}$) can be mitigated by recalculating the generating unit state $I$ and the corresponding generation $P_g$ at hour $t$.

$$\begin{align*}
w(I, P_g) &= \dot{w} + \sum_{i=1}^{NQ} \pi_{it}^g (P_{it} I_{it} - \bar{P}_{it} \bar{I}_{it}) \leq 0. \quad (30)
\end{align*}$$

2) AC Network Security Subproblem: Although line flow violations could be eliminated in the dc network security subproblem, the subproblem formulation (21)–(29) neither considers bus voltage violations nor checks whether there is a feasible distribution of reactive power in power systems. In addition, interactions between real and reactive power constraints are ignored in (21)–(29). Note that the rescheduling of real power controls (e.g., unit state and generation) might indeed help relieve reactive flow violations (e.g., reactive power generating capacity, bus voltage limits), which could not be relieved by adjusting reactive means alone.

For instance, from the economics viewpoint, UC could schedule a cost-effective generating unit to supply real power to remote loads through a long transmission line. However, from the security viewpoint, such UC might result in substantially low voltages at remote buses. Assume that we cannot correct undervoltages by redirecting power flows via control transformer adjustments or rescheduling power generation. Then, violations could be mitigated by our proposed method by committing a new generating unit at a different location and/or adjusting the power generation of existing units at a location near the load center. In order to model the ac network security subproblem, one of the critical elements of power systems, i.e., control transformer with tap-changing and phase-shifting adjustments, ought to be modeled. The detailed transformer model is presented in the Appendix.

Before checking ac network security violations, we make sure that the ac power flow solution is converged based on the commitment and dispatch of units in the master problem. According to the Newton–Raphson method, an ac power flow is converged when real and reactive mismatches at each bus are within an acceptable tolerance $\varepsilon_2$ in Fig. 3.

We define the objective function (31) for minimizing slack variables in this case. By the minimization of these variables in the objective function, we look for a feasible power flow solution.

In matrix form, linearized real and reactive power balance equations are expressed as in (32), where $dP_0$ and $dQ_0$ represent mismatches between scheduled and calculated bus power injections. Note that we need two positive slack variables for an equality constraint. Here, slack variables $MP_{b,1}$ and $MP_{b,2}$ for the real power mismatch at bus $b$, and $MQ_{b,1}$ and $MQ_{b,2}$ for the reactive power mismatch at bus $b$ are nonzero if the power flow solution does not converge. These variables correspond to those in (18). From a physical viewpoint, slack variables represent virtual generators/loads that are added to each bus to eliminate mismatches.

Constraints (33)–(36) represent the limits on real and reactive power generation, transformer tap position, and phase shifter angle, respectively. Note that the constraint (33) applies to all units except that of the slack bus, which is relaxed to compensate for power loss calculation errors. For minimizing the mismatch, bus voltage magnitudes and line flows are not constrained because we are seeking a converged power flow solution based on the Newton–Raphson method.

$$\begin{align*}
\text{Min } w(I, P_g) &= \sum_{i=1}^{NB} (MP_{b,1} + MP_{b,2}) \\
& + \sum_{i=1}^{NB} (MQ_{b,1} + MQ_{b,2}) \\
& \quad [Y^* \Delta P] - [H N E F][\Delta V] \\
& \quad [Y^* \Delta Q] - [M J R S][\Delta T] \\
& \quad [MP_1] - [MP_2] = [dP_0] \quad [dQ_0] \\
& \quad \Delta V = 0 \quad \pi \\
& \quad \Delta Q_{\text{min}} \leq \Delta Q \leq \Delta Q_{\text{max}} \quad \theta, \hat{\theta} \\
& \quad T_{\text{min}} - T_0 \leq \Delta T \leq T_{\text{max}} - T_0 \\
& \quad \gamma_{\text{in}t} - \gamma_0 \leq \Delta \gamma \leq \gamma_{\text{max}} - \gamma_0.
\end{align*}$$

Authorized licensed use limited to: Mississippi State University. Downloaded on January 19, 2010 at 13:54 from IEEE Xplore. Restrictions apply.
We will use an iterative method based on the above formulation to minimize bus power mismatches. The procedure is as follows.

Step 1) Calculate Jacobian matrices and the initial bus mismatch vectors \( \mathbf{dP}_0 \) and \( \mathbf{dQ}_0 \) based on the initial generation schedule, system states, and settings.

Step 2) Use LP to minimize the objective function (31) and calculate changes in system state and control variables \( (\Delta Q, \Delta \delta, \Delta V, \Delta T, \text{and} \Delta \gamma) \).

Step 3) Update state and control variables. Calculate elements of Jacobian matrices and bus mismatch vectors \( \mathbf{dP}_0 \) and \( \mathbf{dQ}_0 \).

Step 4) Use LP to minimize the objective function (31) and calculate changes in system state and control variables. If the difference between current and previous iterative changes is less than a specified threshold, stop the process. Otherwise, go back to Step 3.

The procedure for minimizing bus mismatches is similar to that of solving the ac power flow iteratively.

If the objective function (31) is higher than the specified tolerance \( \varepsilon \) after several iterations, the current unit commitment and generation dispatch cannot provide a feasible ac power flow solution. Thus, a corresponding Benders mismatch violation cut (37) will be formed and added to the master UC problem for the next iteration

\[
\begin{align*}
\min \quad & \mathbf{w}(\mathbf{I}, \mathbf{P}_g) = \mathbf{w} + \sum_{i=1}^{NG} \pi_{it}^d (P_{d,i}^t - \bar{P}_{d,i}^t) \\
& + \sum_{i=1}^{NG} \psi_d^H Q_{i_{\text{max}}} (I_{i} - \bar{I}_{i}) \\
& - \sum_{i=1}^{NG} \psi_d^L Q_{i_{\text{min}}} (I_{i} - \bar{I}_{i}) \leq 0
\end{align*}
\]

where \( \mathbf{w} \) is the current value of (31). A Benders cut is formed only when the violation exists, and there are as many Benders cuts as the number of hours when violations arise. The inclusion of cuts presumes that the new ac power flow solution would converge within its limits by calculating generating unit state \( \mathbf{I} \) and the corresponding generation \( \mathbf{P}_g \) at hour \( t \) in the UC problem.

Once a feasible ac power flow solution is calculated based on the above algorithm, we check the ac network security violations at each hour. This algorithm corresponds to the middle part of Fig. 3.

In order to ensure feasibility, slack variables are added to transmission flow and bus voltage constraints in the subproblem. The slack variables are defined as the amount of constraint violations associated with the given UC state \( \mathbf{I} \) and ED state \( \mathbf{P}_g \) at hour \( t \). Positive and weighted penalty costs of violations are introduced in the objective function of (38).

The prevailing constraints include the equality constraint (39) represented as a matrix for the first-order approximation of ac network, which includes linearized real power balance, reactive power balance, and real power flow equations.

Constraints (40)–(49) represent limits on state and control variables. Constraints (40)–(42) are the real power flow limits on line \( i \). Constraints (43)–(45) represent voltage limits at bus \( b \). Similarly, constraints (46)–(49) represent limits on real power generation, reactive power generation, transformer taps, and phase shifter angles, respectively. Constraint (46) applies to all units except that of slack bus, which is relaxed to pick up loss calculation mismatches

\[
\begin{align*}
\min \quad & \mathbf{w}(\mathbf{I}, \mathbf{P}_g) = \sum_{i=1}^{NL} d_l (C_{l,1} + C_{l,2}) \\
& + \sum_{b=1}^{NB} e_b (C_{b,1} + C_{b,2}) \\
\end{align*}
\]

\[
\begin{bmatrix}
Y \ast \Delta P \\
Y \ast \Delta Q \\
\Delta \mathbf{PL}
\end{bmatrix}
= 
\begin{bmatrix}
H & N & E & F \\
M & J & R & S \\
A & B & C & D
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta V \\
\Delta T
\end{bmatrix}
\]

(49). If network violations persist in this case, a Benders cut (50) associated with the \( n \)th iteration of master problem (UC) is formulated. Accordingly, the current value of transmission flow and bus voltage violations \( \mathbf{w} \) in (38) will be mitigated by applying these cuts to UC, as discussed in Fig. 2.

Once the constraints are linearized, the iterative method is used for solving the optimization problem (38)–(49). If network violations persist in this case, a Benders cut (50) associated with the \( n \)th iteration of master problem (UC) is formulated. Accordingly, the current value of transmission flow and bus voltage violations \( \mathbf{w} \) in (38) will be mitigated by applying these cuts to UC, as discussed in Fig. 2.

\[
\begin{align*}
\min \quad & \mathbf{w}(\mathbf{I}, \mathbf{P}_g) = \mathbf{w} + \sum_{i=1}^{NG} \pi_{it}^d (P_{d,i}^t - \bar{P}_{d,i}^t) \\
& + \sum_{i=1}^{NG} \psi_d^H Q_{i_{\text{max}}} (I_{i} - \bar{I}_{i}) \\
& - \sum_{i=1}^{NG} \psi_d^L Q_{i_{\text{min}}} (I_{i} - \bar{I}_{i}) \leq 0
\end{align*}
\]

Note that cuts that were used in the previous \( n = 1, 2, \ldots, N - 1 \) iterations will also be included as constraints in the current iteration of the master problem. If additional cuts are not obtained based on the solution of subproblem, the iterative process will end.

Mathematically, we can check the convergence of the ac power flow and the security constraints by solving a single subproblem based on (31)–(36) and (40)–(45). However, from a physical viewpoint, we would be interested in determining the reasons that could lead to mismatches (i.e., no converged ac power flow or other system violations). The second alternative approach would fail to provide the reasons. Based on the considerations, we use our proposed sequence of subproblems. At first, we provide a feasible UC that can guarantee a converged
TABLE I
GENERATOR DATA

<table>
<thead>
<tr>
<th>U</th>
<th>Bus No.</th>
<th>Unit Cost Coefficients</th>
<th>Pmax (MW)</th>
<th>Pmin (MW)</th>
<th>Qmax (MVAR)</th>
<th>Qmin (MVAR)</th>
<th>Ini. St. (h)</th>
<th>Min Down (h)</th>
<th>Min Up (h)</th>
<th>Ramp (MW/h)</th>
<th>Start Up (MBtu)</th>
<th>Fuel Price ($/MBtu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1</td>
<td>176.9</td>
<td>13.5</td>
<td>0.1</td>
<td>220</td>
<td>100</td>
<td>50</td>
<td>-40</td>
<td>4</td>
<td>2</td>
<td>55</td>
<td>100</td>
</tr>
<tr>
<td>G2</td>
<td>2</td>
<td>129.9</td>
<td>32.6</td>
<td>0.1</td>
<td>100</td>
<td>10</td>
<td>50</td>
<td>-40</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>200</td>
</tr>
<tr>
<td>G3</td>
<td>6</td>
<td>137.4</td>
<td>17.6</td>
<td>0.1</td>
<td>20</td>
<td>10</td>
<td>50</td>
<td>-40</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>20</td>
</tr>
</tbody>
</table>

TABLE II
BUS DATA

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage-Max (pu)</th>
<th>Voltage-Min (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
<td>0.95</td>
</tr>
<tr>
<td>2</td>
<td>1.15</td>
<td>0.85</td>
</tr>
<tr>
<td>3</td>
<td>1.15</td>
<td>0.85</td>
</tr>
<tr>
<td>4</td>
<td>1.05</td>
<td>0.91</td>
</tr>
<tr>
<td>5</td>
<td>1.15</td>
<td>0.85</td>
</tr>
<tr>
<td>6</td>
<td>1.15</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Fig. 4. Six-bus system.

c power flow. The ac power flow solution is then used as a reasonable initial value for the second subproblem (38)–(49), which could speed up the convergence of the constrained power network solution.

We compared these two approaches and, as expected, obtained the same solution for feasible cases. In our SCUC package, we provide the user with an option for applying either one of the two approaches.

III. CASE STUDIES

We apply two case studies consisting of a six-bus system and the IEEE 118-bus system to illustrate the performance of SCUC with ac constraints.

A. SIX-BUS SYSTEM

The six-bus system, depicted in Fig. 4, has three units, five transmission lines, and two tap-changing transformers. The characteristics of generators, buses, transmission lines, and tap-changing transformers and the hourly load distribution over the 24-h horizon are given in Tables I–V, respectively.

The unit shutdown cost is negligible and assumed to be zero in this case. In order to discuss the efficiency of the proposed approach in detail, we consider the following five cases:

- Case 0: Base case without any network constraints.
- Case 1: Consider dc transmission flow violations.
- Case 2: Consider ac transmission flow violations.
- Case 3: Consider bus voltage violations.
- Case 4: Consider ac network (transmission flow and bus voltage) violations.

Case 0: We calculate the UC solution in the master problem by excluding transmission and voltage constraints. The commitment schedule is shown in Table VI, in which 1 and 0 represent ON/OFF states of units at different hours, and hour 0 represents the initial condition. The daily operating cost is $96 882.23. In this case, the expensive units 2 and 3 are not committed at certain hours in order to minimize the operating cost.

TABLE III
TRANSMISSION LINE DATA

<table>
<thead>
<tr>
<th>Line No.</th>
<th>From Bus</th>
<th>To Bus</th>
<th>R (pu)</th>
<th>X (pu)</th>
<th>Flow Limit (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0.0050</td>
<td>0.170</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
<td>0.0030</td>
<td>0.258</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>4</td>
<td>0.0070</td>
<td>0.197</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>0.0020</td>
<td>0.140</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>6</td>
<td>0.0005</td>
<td>0.018</td>
<td>100</td>
</tr>
</tbody>
</table>

TABLE IV
TAP-CHANGING TRANSFORMER DATA

<table>
<thead>
<tr>
<th>Transformer No.</th>
<th>From Bus</th>
<th>To Bus</th>
<th>X (pu)</th>
<th>Tap Max</th>
<th>Tap Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>2</td>
<td>3</td>
<td>0.037</td>
<td>0.98</td>
<td>0.95</td>
</tr>
<tr>
<td>T2</td>
<td>4</td>
<td>5</td>
<td>0.037</td>
<td>0.98</td>
<td>0.95</td>
</tr>
</tbody>
</table>

TABLE V
HOURLY LOAD

<table>
<thead>
<tr>
<th>H</th>
<th>Pd (MW)</th>
<th>Qd (MVAR)</th>
<th>H</th>
<th>Pd (MW)</th>
<th>Qd (MVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>175.19</td>
<td>50.37</td>
<td>13</td>
<td>242.18</td>
<td>69.63</td>
</tr>
<tr>
<td>2</td>
<td>165.15</td>
<td>47.48</td>
<td>14</td>
<td>243.60</td>
<td>70.03</td>
</tr>
<tr>
<td>3</td>
<td>158.67</td>
<td>45.62</td>
<td>15</td>
<td>248.86</td>
<td>71.55</td>
</tr>
<tr>
<td>4</td>
<td>154.73</td>
<td>44.49</td>
<td>16</td>
<td>255.79</td>
<td>73.54</td>
</tr>
<tr>
<td>5</td>
<td>155.06</td>
<td>44.58</td>
<td>17</td>
<td>256.00</td>
<td>73.60</td>
</tr>
<tr>
<td>6</td>
<td>160.48</td>
<td>46.14</td>
<td>18</td>
<td>246.74</td>
<td>70.94</td>
</tr>
<tr>
<td>7</td>
<td>173.39</td>
<td>49.85</td>
<td>19</td>
<td>245.97</td>
<td>70.72</td>
</tr>
<tr>
<td>8</td>
<td>177.60</td>
<td>51.06</td>
<td>20</td>
<td>237.35</td>
<td>68.24</td>
</tr>
<tr>
<td>9</td>
<td>186.81</td>
<td>53.71</td>
<td>21</td>
<td>237.31</td>
<td>68.23</td>
</tr>
<tr>
<td>10</td>
<td>206.96</td>
<td>59.50</td>
<td>22</td>
<td>232.67</td>
<td>66.89</td>
</tr>
<tr>
<td>11</td>
<td>228.61</td>
<td>65.73</td>
<td>23</td>
<td>195.93</td>
<td>56.33</td>
</tr>
<tr>
<td>12</td>
<td>236.10</td>
<td>67.88</td>
<td>24</td>
<td>195.60</td>
<td>56.23</td>
</tr>
</tbody>
</table>

- Case 3: Consider bus voltage violations.
- Case 4: Consider ac network (transmission flow and bus voltage) violations.
at hours 12, 20, 21, and 22, as shown in Table VII. Here, T-Cuts refer to cuts for transmission flow violations. In addition, Table VII shows that dc flow violations are mitigated after introducing Benders cuts in SCUC. In order to satisfy transmission flow limits, SCUC has committed unit 2 at hours 12, 20, 21, and 22 with a daily operating cost of $98,354.93. The cost, shown in Table VIII, is higher than that of Case 0.

**Case 2:** When we consider ac transmission flow constraints, based on the UC schedule in Case 0, violations occur on line 1–4 at hours 12, 20, and 21, as shown in Table IX. Note that unlike Case 1, we do not encounter a flow violation on line 1–4 at hour 22, which is due to the interaction between real and reactive power flows (39). Table X shows that once we introduce ac Benders cuts, unit 2 is committed additionally at hours 12, 20, and 21 to mitigate flow violations on line 1–4. Table X shows that the daily operating cost of SCUC is $97,970.34, which is cheaper than that in Case 1.

**Case 3:** When we consider voltage constraints based on the Case 0 results, there will be voltage violations on bus 4 at hours 10, 23, and 24. Table XI shows that as we execute the SCUC, voltage violations are mitigated based on Benders cuts. Similarly, V-Cuts refer to cuts for bus voltage violations. SCUC has a daily operating cost of $99,471.16, as shown in Table XII. In order to eliminate voltage violations, unit 2 is committed at hours 10, 23, and 24. The expensive unit 2 maintains its ON state at hours 10 and 12 for satisfying unit constraints such as ramp rate and minimum up/down time limits. Furthermore, unit 2, which has an expensive startup cost, will remain ON at hours 20 through 22, which results in the OFF state of unit 3 at those hours.

**Case 4:** We find in this case that the transmission and voltage violations in Cases 2 and 3 are eliminated simultaneously by executing the SCUC with ac network security constraints. Table XIII shows the daily operating cost of SCUC at $99,568.13, which is higher than those of Cases 0 through 3 because of the additional commitment of units 2 and 3.

### A. IEEE 118-Bus System

A modified IEEE 118-bus system is used to study the SCUC with ac network constraints. The system has 54 units, 186 branches, 14 capacitors, nine tap-changing transformers, and 91 demand sides. The system is divided into three zones. The peak load of 3733 MW occurs at hour 15. The network topology is shown in Fig. 5, and the test data for the 118-bus system are given in motor.ece.iit.edu/data/SCUC_118test.xls.

Table XIV presents the base case SCUC without network constraints with a daily operating cost of $849,291.48. The execution time is 30 sec on a 1.8-GHz personal computer. In this

| TABLE VI |
| UC WITHOUT TRANSMISSION AND VOLTAGE CONSTRAINTS |
| Daily Cost = $96,882.23 |

<table>
<thead>
<tr>
<th>U</th>
<th>Hours (0-24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
</tr>
<tr>
<td>2</td>
<td>1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1</td>
</tr>
<tr>
<td>3</td>
<td>1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1</td>
</tr>
</tbody>
</table>

| TABLE VII |
| FLOW ON LINE 1–4 AT VIOLATED HOURS (MW) |

<table>
<thead>
<tr>
<th>Hours</th>
<th>12</th>
<th>20</th>
<th>21</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Cuts</td>
<td>103.92</td>
<td>104.51</td>
<td>104.49</td>
<td>102.31</td>
</tr>
<tr>
<td>T-Cuts</td>
<td>77.32</td>
<td>77.77</td>
<td>77.75</td>
<td>76.12</td>
</tr>
</tbody>
</table>

| TABLE VIII |
| SCUC WITH DC TRANSMISSION CONSTRAINTS |
| Daily Cost = $98,354.93 |

<table>
<thead>
<tr>
<th>U</th>
<th>Hours (0-24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
</tr>
<tr>
<td>2</td>
<td>1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1</td>
</tr>
<tr>
<td>3</td>
<td>1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1</td>
</tr>
</tbody>
</table>

| TABLE IX |
| FLOW ON LINE 1–4 AT VIOLATED HOURS (MW) |

<table>
<thead>
<tr>
<th>Hours</th>
<th>12</th>
<th>20</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Cuts</td>
<td>102.16</td>
<td>102.72</td>
<td>102.7</td>
</tr>
<tr>
<td>T-Cuts</td>
<td>84.29</td>
<td>84.91</td>
<td>84.89</td>
</tr>
</tbody>
</table>

| TABLE X |
| SCUC WITH AC TRANSMISSION CONSTRAINTS |
| Daily Cost = $97,970.34 |

<table>
<thead>
<tr>
<th>U</th>
<th>Hours (0-24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
</tr>
<tr>
<td>2</td>
<td>1 1 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1</td>
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<tr>
<td>3</td>
<td>1 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1</td>
</tr>
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</table>

| TABLE XI |
| VOLTAGES ON BUS 4 AT VIOLATED HOURS (PU) |

<table>
<thead>
<tr>
<th>Hours</th>
<th>10</th>
<th>23</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Cuts</td>
<td>0.8867</td>
<td>0.8915</td>
<td>0.8914</td>
</tr>
<tr>
<td>V-Cuts</td>
<td>0.9499</td>
<td>0.9587</td>
<td>0.9586</td>
</tr>
</tbody>
</table>

| TABLE XII |
| SCUC WITH AC VOLTAGE CONSTRAINTS |
| Daily Cost = $99,471.16 |

<table>
<thead>
<tr>
<th>U</th>
<th>Hours (0-24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
</tr>
<tr>
<td>2</td>
<td>1 1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
</tr>
<tr>
<td>3</td>
<td>1 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1</td>
</tr>
</tbody>
</table>

| TABLE XIII |
| SCUC WITH AC NETWORK SECURITY CONSTRAINTS |
| Daily Cost = $99,568.13 |

<table>
<thead>
<tr>
<th>U</th>
<th>Hours (0-24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
</tr>
<tr>
<td>2</td>
<td>1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1</td>
</tr>
<tr>
<td>3</td>
<td>1 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1</td>
</tr>
</tbody>
</table>

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system, economical units (such as 4–5, 10–11, 43, and 45) are used as base units and expensive units (such as 1–3, 6–9, and 46–52) are not committed at all. The remaining units are committed accordingly to satisfy hourly load demands. Then, the subproblem represented by (31)–(36) is solved, and results show that a converged power flow is obtained based on the given UC in Table XIV. In order to check if there are any ac violations, we run the hourly network security check subproblem (38)–(49) based on the initial UC in Table XIV. In this example, there are eight transmission flow violations on line 82–83 at hours 1–7 and 24. In addition, the voltage at bus 111 is less than its lower limit of 1.0 pu (about 0.98 pu) at hours 9–23. The initial 23 Benders cuts (eight T-Cuts plus 15 V-Cuts) for violated hours are introduced to the master problem. After three iterations, the new UC solution is presented in Table XVI with a daily operating cost of $851,274.38. The execution time is around 100 sec on a 1.8-GHz personal computer.

In Table XVI, unit 39 in zone 3 is decommitted at hours 1–10 and 24 to mitigate the violation by reducing the supply of power to zone 2. Correspondingly, units 21, 24, 25, 28, 34, 37, and 53 in zone 2 are committed at certain hours (e.g., 1–6, 9–19, and 24) to compensate for the reduced supply from zone 3 and to satisfy physical constraints such as the minimum down time of unit 39 and the minimum up time of unit 24. Table XV shows that transmission flow violations are eliminated based on the new UC solution. In addition, the voltage at bus 111 is increased to about 1.04 pu, which is within the voltage limits of 1.0 and 1.06 pu. This adjustment is due to the commitment of unit 51 at bus 111 between hours 9 and 23.

We applied the proposed approach successfully to a practical system with 698 generators, 4672 buses, 5917 branches, 2230 loads, 228 shunt capacitors, and 76 control transformers. This case was also tested on a 1.8-GHz personal computer. The CPU time in this case depended on the load level and the number of constraints considered in the example. A typical CPU time without network constraints was about 10 min, with dc network constraints was 30 min, and with ac network constraints was 100 min. The total number of iterations for SCUC with ac network constraints was six. The CPU time in such cases could be reduced by parallel processing. The solution results in this study indicate that the proposed ac method is applicable to the day-ahead SCUC calculation of large-scale power systems.

Fig. 5. One-line diagram of IEEE 118-bus system.

IV. CONCLUSION

In order to ensure the secure operation of a power system and supply the customers with high-quality electricity, the ISO strives to maintain transmission flows and bus voltages within their permissible limits. In this paper, an SCUC approach is presented which could be utilized by an ISO to schedule the power
generation economically, while guaranteeing the power network security. The tests on the six-bus and the IEEE 118-bus systems showed the effectiveness of the proposed approach in a restructured electricity market. The application of Benders decomposition divides the original SCUC optimization into a UC master problem and hourly network security check subproblems. The subproblem generates Benders cuts corresponding to network violations, which are added to the master problem for rescheduling UC. The salient feature of the proposed approach is that it considers transmission flow and bus voltage constraints as well as the interaction between real and reactive power in a market operation framework. The approach based on Benders decomposition techniques is viewed as a trade-off between economics, security and calculation speed.

**Appendix**

**Control Transformer Model:** A control transformer for SCUC with tap-changing and phase-shifting capabilities is modeled in this section. The transformer model connecting buses j and m is shown in Fig. 6, which is used for calculating the elements of Jacobian matrix. In this figure, j is the tap side, m is the nontap side, \( k_T \) is the complex off-nominal ratio of phase shifting transformer \( k_T = T_{jm} e^{j\phi_{jm}} \), and \( \gamma_T \) is the series admittance in per unit based on the nominal turn ratio. Accordingly, \( \gamma_T = \gamma_{jm} = g_{jm} + jb_{jm} = (1/r_{jm} + jx_{jm}). \)

For the assumed direction of currents, we have

\[
\begin{align*}
\dot{V}_j & = k_T V_{jm} \\
\dot{V}_{jm} & = \gamma_T (V_m - V_{jm})
\end{align*}
\]

Then, current \( \dot{I}_m \) is given by

\[
\dot{I}_m = \gamma_T (V_m - V_{jm}).
\]
From (A1) and (A4), we get
\[
\dot{I}_m = y_T (V_m - V_{j0}) = y_T (V_m - (1/k_T) \dot{V}_{j0}) = y_T V_m - (y_T/T_{jm}) e^{-j\gamma_{jm}} V_j.
\] (A5)

From (A1)–(A5), we get
\[
I_j = I_{j0} (V_{j0}/V_j)^* = I_{j0} (1/k_T)^* = I_{j0} (1/T_{jm}) e^{j\gamma_{jm}} = -(y_T/T_{jm}) e^{j\gamma_{jm}} V_m + (y_T/T_{jm}^2) V_j.
\] (A6)

So, we have the following matrix for (A5) and (A6):
\[
\begin{bmatrix}
\dot{I}_j \\
I_m
\end{bmatrix} =
\begin{bmatrix}
Y_{L_{jj}} & Y_{L_{jm}} \\
Y_{L_{mj}} & Y_{L_{mm}}
\end{bmatrix}
\begin{bmatrix}
\dot{V}_j \\
V_m
\end{bmatrix}.
\] (A7)

From (A1)–(A6), self-admittances are
\[
Y_{L_{jj}} = (g_{jm} + jb_{jm})/T_{jm}^2 = g_{jm}/T_{jm} + jb_{jm}/T_{jm}^2 = GL_{jj} + jBL_{jj}
\]
\[
Y_{L_{mm}} = y_T = g_{jm} + jb_{jm} = GL_{mm} + jBL_{mm}.
\]

Mutual admittances are
\[
Y_{L_{jm}} = -(1/T_{jm})(g_{jm} + jb_{jm}) e^{j\gamma_{jm}} = -(1/T_{jm})(g_{jm} + jb_{jm})(\cos \gamma_{jm} + j\sin \gamma_{jm}) = Y_{L_{mj}} = -(1/T_{jm})(g_{jm} \cos \gamma_{jm} + jb_{jm} \sin \gamma_{jm}) + j\gamma_{jm} = GL_{jm} + jBL_{mj}
\]
where
\[
GL_{jj} = g_{jm}/T_{jm}^2 \\
BL_{jj} = b_{jm}/T_{jm}^2 \\
GL_{jm} = -(g_{jm} \cos \gamma_{jm} + jb_{jm} \sin \gamma_{jm})/T_{jm} \\
BL_{jm} = -(b_{jm} \cos \gamma_{jm} + g_{jm} \sin \gamma_{jm})/T_{jm} \\
GL_{mj} = -(g_{jm} \cos \gamma_{jm} + jb_{jm} \sin \gamma_{jm})/T_{jm} \\
BL_{mj} = -(b_{jm} \cos \gamma_{jm} + g_{jm} \sin \gamma_{jm})/T_{jm} \\
GL_{mm} = g_{jm} \\
BL_{mm} = b_{jm}.
\]

If we consider line charging, we get
\[
BL_{jj} = b_{jm}/T_{jm}^2 + k_{ch}/2 \\
BL_{mm} = b_{jm} + k_{ch}/2.
\]

According to (A7), real and reactive bus injections are formulated as follows:

Real power at bus j is
\[
P_j = V_j^2 G_{jj} + V_j \sum_{m \in S_j} V_m G_{jm} \cos(\delta_j - \delta_m) + B_{jm} \sin(\delta_j - \delta_m).
\] (A8)

Reactive power at bus j is
\[
Q_j = -V_j^2 B_{jj} + V_j \sum_{m \in S_j} V_m G_{jm} \sin(\delta_j - \delta_m) - B_{jm} \cos(\delta_j - \delta_m)
\]
where
\[
G_{jj} = \sum_{m \in S_j} GL_{jj} \\
B_{jj} = \sum_{m \in S_j} BL_{jj} \\
G_{jm} = \sum_{m \in S_j} GL_{jm} \\
B_{jm} = \sum_{m \in S_j} BL_{jm}
\]
where \(S_j\) represents the set of buses connected to bus j, and \(S_{jm}\) represents the set of branches that link buses j and m.

The partial derivative of (A8) and (A9) with respect to bus phase angle \(\delta\), voltage magnitude \(V\), transformer tap \(T\), and phase shifter angle \(\gamma\) are represented as follows:
\[
H = \frac{\partial P}{\partial \delta} \\
N = \frac{\partial P}{\partial V} \\
E = \frac{\partial P}{\partial T} \\
F = \frac{\partial P}{\partial \gamma}
\]
\[
M = \frac{\partial Q}{\partial \delta} \\
J = \frac{\partial Q}{\partial V} \\
R = \frac{\partial Q}{\partial T} \\
S = \frac{\partial Q}{\partial \gamma}.
\]

The Jacobian submatrices (\(H, N, E, F, M, J, R\), and \(S\)) provide a linearized relationship between small changes in the system state and control variables \((\Delta \delta, \Delta V, \Delta T, \text{ and } \Delta \gamma)\) with respect to small changes in real and reactive power mismatches. The complex power flow on line j-m is given as
\[
SL_{jm} = PI_{jm} + jQL_{jm} = V_j^2 I_j^* = V_j^2 (YL_{jj} V_j + YL_{jm} V_m)^* = V_j^2 (GL_{jj} + jBL_{jj}) + V_j V_m (\cos(\delta_j - \delta_m) + j \sin(\delta_j - \delta_m)) (GL_{jm} - jBL_{jm}).
\] (A10)

Separating real and imaginary parts, the real power flow from bus j to m is
\[
PI_{jm} = V_j^2 GL_{jj} + V_j V_m (GL_{jm} \cos(\delta_j - \delta_m) + BL_{jm} \sin(\delta_j - \delta_m)).
\] (A11)
The reactive power flow from bus j to m
\[ Q_{L \text{jm}} = -V_j^2 B_{L \text{jj}} + V_j V_m (G_{L \text{jm}} \sin(\delta_j - \delta_m) - \delta_m) - B_{L \text{jm}} \cos(\delta_j - \delta_m)). \]  
(A12)

From (A11), we calculate partial derivatives of real power flow with respect to bus phase angle \( \delta \), voltage magnitude \( V \), transformer tap \( T \), and phase shifter angle \( \gamma \) as follows:

\[
\begin{align*}
A &= \frac{\partial Q_{L \text{jm}}}{\partial \delta}, & B &= \frac{\partial Q_{L \text{jm}}}{\partial V}, & C &= \frac{\partial Q_{L \text{jm}}}{\partial T}, & D &= \frac{\partial Q_{L \text{jm}}}{\partial \gamma}.
\end{align*}
\]

Similarly, Jacobian submatrices \((A, B, C, D)\) give the linearized relationship between small changes in system state and control variables \((\delta_j, \delta_m, V, T, \gamma)\) with respect to small changes in real power transmission flows. According to these derivations, we calculate the elements of Jacobian matrix in linearized power flow equations.

REFERENCES


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