Miscellaneous Appendices
DECIBEL (dB) MEASURE AND ORDERS-OF-MAGNITUDE

Within any engineering context an order-of-magnitude reference is necessary. For the electrical sciences the measure is in terms of the decibel (dB).

Aside from the history and a more extensive overlook (https://en.wikipedia.org/wiki/Decibel) the measure relates to orders of magnitude of power. A factor of 10 change in power corresponds to 10 dB, i.e.

\[
10 \text{ dB} = 10 \times \log_{10} \left( \frac{p_2}{p_1} \right) = 10 \quad \text{(base 10 logarithms)}
\]

Similarly a change by a factor of 100 will be 20 dB since

\[
20 \text{ dB} = 10 \times \log_{10} \left( \frac{p_2}{p_1} = 100 \right) \quad \text{etc.}
\]

So dB measure is the (exponent of 10 to a power) × 10. The logarithm is (by definition) the exponent.

Take note that 0 dB corresponds to a factor of 1.0 since anything to the 0th power = 1.

Of special interest are factors of 2, for which

\[
10 \times \log_{10} \left( \frac{p_2}{p_1} = 2 \right) = 3 \text{ dB}
\]

A factor of \( \frac{1}{2} \) (the half-power level)

\[
10 \times \log_{10} \left( \frac{1}{2} \right) = -3 \text{ dB}
\]

Take note that:

\[
10 \log (2^2) = 2 \times 10 \log (2) = 6 \text{ dB}
\]

\[
10 \log (2^{-2}) = -2 \times 10 \log (2) = -6 \text{ dB}
\]

\[
10 \log (2^3) = 3 \times 10 \log (2) = 9 \text{ dB} \quad \text{etc.}
\]

It is possible to construct any integer measure in dB from these simple relationships.

For example a factor of

\[
5 = 10/2 = 10 \text{ dB} - 3 \text{ dB} = 7 \text{ dB}
\]

\[
2.5 = 5/2 = 7 \text{ dB} - 3 \text{ dB} = 4 \text{ dB}
\]

So for integer values of dB there is no need for a calculator. From the above, five of the factor equivalents are now known. The others are
The same order-of-magnitude factor applies to relative measures. The most common is the power level relative to 1 watt \( \equiv \text{dBW} \),

For example: 100W corresponds to 20 dBW since 
\[
10 \log \left( \frac{100\text{W}}{1.0\text{W}} \right) = 20
\]

It is also not uncommon to apply the relative measure of power relative to a milli-Watt \( \equiv \text{dBm} \).

For example 0.1W = 100 mW corresponds to 20 dBm. It also corresponds to -10dBW.

Most transfer functions originate as transfer ratios for voltage or current. Since power relates to these quantities as the square an increase in a factor of 10 in the voltage corresponds to an increase by a factor of 100 in power

So they will have the prefix of 20 instead of 10 concerning dB measure, i.e.

\[
20 \text{ dB} = 20 \times \log \left( \frac{v_2}{v_1} = 10 \right)
\]

\[
40 \text{ dB} = 20 \times \log \left( \frac{v_2}{v_1} = 100 \right) \quad \text{etc.}
\]

So the same context for dB usage as a measure of factor of 10 is kept, except the mathematics is multiplied by a factor of 2.

Of special interest is the factor \( \sqrt{2} \). This factor corresponds to 3dB since

\[
20 \log \sqrt{2} = 20 \times (1/2) \log 2 = 10 \log 2 = 3\text{dB}
\]

The roll-off corner for which \( |v_2/v_1| = 1/\sqrt{2} \) then corresponds to -3dB, and is so designated. This roll-off corner also corresponds to the half-power (roll-off) point.
THE STORY OF THE DECIBEL

The history for the unit of measure called the decibel (dB) is not well known. We know of it as a log10 (decade) relative measure of power amplitude. But its origin comes from far back in the dark ages of early European history.

Europe is replete with churches. Churches have bells. And once upon a time the profession of bellringer was an option, often tasked to those who were afflicted by scoliosis (hunchbacks) and could not work in the fields. I would expect that most of you have heard of the hunchback of Notre Dame.

It was the year 1491, Kiev, Ukraine. Fortunately we have faculty from that part of the world and so we are privy to the true story of the dB (which not even Wikipedia knows). Our faculty from the Ukraine tells the story of the great bell tower of Kiev. Sadly the story begins with the passing of the bellringer for the cathedral. And thus the bishop sent forth a call for someone to take his place.

To his great surprise a young, strongly built hunchback with no arms applied. 'Indeed I doth appreciate thy inquiry young man, but how canst thou possibly ring the bells?' the bishop asked.

'Forssoth, let us go up to the bell tower your Grace and I will show thee,' said the young man.

The loft for the bell tower was large and spacious and the young man could move quickly and easily. He ran straight at the bell for middle C and struck it head-on with his broad forehead. And the bell rang forth with one of the sweetest tones the bishop had ever heard. 'Forssoth, I had my doubts, but that doth be the most beautiful sound I have ever heard. Thou art hired,' said the bishop.

The bell ringer could ring all of the bells but he particularly liked the rank of bells that were one tenth the size of the great boomer. And the bishop knew this and usually made his requests accordingly.

But one September day, on the feast of St. Volodymyr, the burghers of Kiev called for the church to ring forth with the mightiest of bells. And the bellringer complied. But on the toll of the last great bell he knocked himself into a daze and staggered aside and fell thru one of the sound portals and to his death on the courtyard below.

The people all rushed to the courtyard. 'Forssoth, who doth be this fellow?' They cried, for none of them knew. The bishop came up and said, 'Alas, I knowest not his name,' he cried. 'But indeed his face doth ring a bell.'

So the call went forth again. And much to the bishop's surprise another young man of like physical strength and features stepped forward. 'I doth be the brother of thy deceased bellringer,' he said. 'And I wouldst ask to ring the bells one last time in my brother's memory'.

'Thou hath my blessings,' said the bishop. And indeed, I would honor thy request this very day, for it is the day of the Lord, and we must call the penitent to the church. Forssoth, let us proceed to the tower loft.'

'So this be where my brother lived', said the young man. 'And how didst he ring the bells so sweetly?'

The bishop explained. The young man looked doubtful. But he said, 'I willst do this in memory of my brother', and he ran at the large deep-toned lower C and sure enough, the bell rang forth with a fine clear deep sound. But it also was a little much for the young man. And in a daze he staggered aside and fell out the same portal thru which his deceased brother had fallen, and to his death on the courtyard below.

The people gathered. The bishop dashed down the stairs to the courtyard. 'Forssoth, who was this man?' the people cried. 'I never kneweth his name. But he doth be a dead ringer for his brother,' said the bishop.

And the people were sad. 'Forssoth, we shouldst bury these honored young men in the church cemetery,' they said.

The bishop said 'Alas, I had a hunch back then I shouldest have pursued the identity of the first young man. But, forssoth, his favorite rank of bells were the set that was one-tenth the size of the mightiest rank, so we will write the latin name 'decibel' on his tombstone.

But there was not enough coinage in the church treasury to pay the surly stonecutter for carving both tombstones, so the names were abbreviated to +3dB and -3dB.

I bet you think I am making this all up.

Go to the old churchyard of St. Sophia in Kiev and see for yourself.  -RW
COMPLEX ARITHMETIC, IMPEDANCE AND ADMITTANCE

Complex numbers are of the form

\[ z = x + jy = (r \cos \phi) + j(r \sin \phi) \]
\[ z = re^{j\phi} = r \angle \phi \]

linear

phasor

The magnitude and phase of a complex number identify with these according to

\[ r = \sqrt{x^2 + y^2} \]
\[ \phi = \tan^{-1}(y/x) \]

If two complex number are added they should both be in the linear form, i.e.

\[ z_1 + z_2 = (x_1 + jy_1) + (x_2 + jy_2) = (x_1 + x_2) + j(y_1 + y_2) \]

The reciprocal of a complex number is best accomplished when it is in the phasor form

\[ \frac{1}{z_1} = \frac{r_1}{r_1 e^{j\phi_1}} = \frac{1}{r_1} e^{-j\phi_1} = g_1 e^{-j\phi_1} \]

same as

\[ \frac{z_1}{z_1} = \frac{1}{r_1 \angle \phi_1} = \frac{1}{r_1} \angle(-\phi_1) = g_1 \angle(-\phi_1) \]

So if one complex number is divided by another the algebra is best accomplished by the phasor form

\[ \frac{z_1}{z_2} = \frac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \frac{r_1}{r_2} \angle(\phi_1 - \phi_2) = r_1 g_2 \angle(\phi_1 - \phi_2) \]

If two complex numbers are multiplied the algebra is best accomplished if they are in the phasor form, but formulae for the linear form also exist.

\[ z_1 z_2 = r_1 \angle \phi_1 \times r_2 \angle \phi_2 = (r_1 \times r_2) \angle(\phi_1 + \phi_2) \]
\[ z_1 z_2 = (x_1 + jy_1) \times (x_2 + jy_2) = (x_1x_2 - y_1y_2) + j(x_1y_2 + x_2y_1) \]

when evaluating a circuit in terms of impedance (Z) and admittance (Y) it is necessary to take reciprocals in as much as \( Y = 1/Z \). The relationships are
\[ Z = R + jX = |Z| \angle \phi \quad Y = \left(\frac{1}{|Z|}\right) \angle -\phi = G + jB \]

Where \( \phi = \tan^{-1}(X/R) = -\tan^{-1}(B/G) \)

Take note that, for the linear form, the magnitudes of the conductance \((G)\) and the susceptance \((B)\) have the same proportional relationship as do the resistance \((R)\) and the reactance \((X)\). The signs of the imaginary term are opposite.