CHAPTER 18. LINEAR ACTIVE FILTERS AND FREQUENCY PROFILES

18.1 INTRODUCTION

All circuits have a frequency response, characterized by their connected set of reactive components and devices and their mix of intrinsic time constants. In many instances it is desired that an analysis of the frequency character of a circuit be accomplished. But in other instances it is desired to adjust the circuit's frequency behavior and define or redefine the frequency profile of the circuit.

In most instances a frequency profile is defined in terms of amplitude ‘pass’ functions. There are four basic types:

1. low-pass
2. high-pass
3. band-pass
4. band-stop

Unless semiconductor components dominate the circuit its time constants should be expected to relate to the \( L, C, \) and \( R \) components. In this respect, intermediate frequency and RF (radio-frequency) profiles may be defined by pF or fF capacitances and \( \mu H \) inductances. Profiles also may be defined by RLC resonance networks and the placement of poles and zeros. High RF profiles usually need to accommodate the effects of parasitic components such as leakage paths, wiring inductances, and fringing capacitances. Frequency profiles in the UHF range (300MHz to 3GHz) or higher will relate to microstrips, resonant cavities or artificial crystals for realizations of desired frequency responses.

At lower frequencies, typically associated with audio systems or other biological interface systems, many components may be large and cumbersome. Consequently techniques have been developed in which amplification loops are used to define profiles. These circuit topologies are usually called active filters. Active filters are also used at higher frequencies in order to be able to deploy the function on an integrated circuit, where all components will be small in both value and size.

An active filter is a frequency-responsive network driven by one or more active drivers. The typical driver typically is an opamp or one of its cousins. Often the circuit can then be created in an integrated-circuit form in which the only frequency-dependent components are capacitances.

The integrated circuit active filter is also appropriate to RF (radio frequencies) as a means to either avoid or to accommodate the parasitic wiring capacitances and inductances.

A frequency profile is generically defined by a transfer function \( T(s) = |T(s)|e^{j\phi(s)} \) in the s-domain for which \( s = j\omega \). And as identified by chapter 5, the graphical representations of \( |T(s)| \) and \( \phi(s) \) are called Bode magnitude and Bode phase plots, respectively, after their originator, Hendrik Bode (1905-1982).
The order of the profile $T(s)$ is defined by the number of frequency-dependent components. All profiles will have a numerator $N(s)$ and a denominator $D(s)$

$$T(s) = \frac{N(s)}{D(s)} \quad (18.1-1)$$

for which the numerator zeros are called zeros and the denominator zeros are called poles. Poles are associated with roll-off of the frequency profile. Zeros will have the effect of ‘roll-upward’. All transfer functions have the same number of zeros as they do poles. The effects of poles are almost always visible. The effects of zeros may be less visible since many zeros will fall either at zero or at infinity.

A great deal of attention has been given to the art of frequency profiling, for which placement of poles and zeros is of critical importance to the character of the profile. Therefore many profile options will be identified in terms of names of their benefactor or in terms of mathematical descriptors. We will not attempt to survey all of the different types except to say that there are enough to satisfy most of the discriminating engineering requirements. Usually the different types will exist in tabulated forms.

For simplicity and conservation of context the table values for profiles of interest can be assumed and then we will load and go, provided we know how to rescale and convert to the frequencies of interest and have enough perspective to make a judicious choice.

**Bilinear circuits:** The bilinear circuits are the simplest category of frequency profile. The transfer function $T(s)$ is a ratio of two linear functions, otherwise called a bilinear form. The general form for the bilinear function is

$$T(s) = K \frac{s + z}{s + p} = K \frac{1 + s/z}{1 + s/p} \quad (18.1-2)$$

Table 18.1-2 shows the magnitude response for the two single-time-constant types of bilinear response (low-pass and high-pass) and their accompanying phase responses. The roll-off slope is of the form of $-20\text{dB/decade}$ (which is the same as a ratio of $-1/1$). The roll-off slope on the log-log plot is linear since for $\omega > 10 \times p$, the magnitude of the denominator becomes

$$|D(s)| = \sqrt{1 + (\omega/p)^2} \rightarrow \omega/p \quad \text{as } \omega \gg p$$
Bode magnitude plots are usually represented as log-log scales with the frequency axis of the form of a decade scale and the amplitude axis of the form of a magnitude scale in dB. The zeros are usually at either infinity or zero (which is typical) and beyond the range of a finite Bode plot.

For a more emphatic roll-off, higher-order transfer functions are required. Assuming appropriate placement of poles and zeros roll-off becomes as much as \( \sim n \times (-20 \text{dB/dec}) \), where \( n \) = number of poles,

\[
|T(s)\rightarrow\infty|\rightarrow 0.
\]

**Biquadratic circuits:** The simplest categories of frequency profiles are (1) first-order forms and (2) second-order forms. The second-order are profiles are called *biquadratic* forms. The biquadratic form yields a complete set of pass-band profiles as represented by Table 18.1-2. The general form of the biquadratic function is

\[
T(s) = K \frac{n_2 s^2 + n_1 s + n_0}{s^2 + s/\omega_o/Q + \omega_o^2} = K' \frac{1 + s/a + s^2/b}{1 + s/\omega_o Q + s^2/\omega_o^2} \quad (18.1\text{-}3)
\]

Table 18.1-2 shows the types of magnitude response for different numerator functions. For convenience and definition of the key profile parameters, the denominator function is always of the form
Equation (18.1-4) is the preferred form for the quadratic denominator since it relates to a characteristic (resonance) frequency $\omega_0 = 2\pi f_0$ and a ‘quality factor’ $Q$. The band-pass function, for which $n_2 = 0$ and $n_0 = 0$, will have peak magnitude at $f_0$ and will be symmetric about $f_0$ on the decade frequency scale. Low-pass and high-pass quadratic functions will also show a profile peak if $Q > 1/\sqrt{2}$, although it will not be very pronounced until $Q > 2$.

<table>
<thead>
<tr>
<th>Type</th>
<th>(Amplitude normalized) transfer function</th>
<th>Magnitude Profile</th>
</tr>
</thead>
</table>
| (a) Low-pass| $T(s) = \frac{\omega_0^2}{s^2 + s \omega_0/Q + \omega_0^2}$  
  ($Q = 4$ represented) | ![Low-pass Transfer Function](image) |
| (b) Band-pass | $T(s) = \frac{s \omega_0/Q}{s^2 + s \omega_0/Q + \omega_0^2}$  
  ($Q = 4$ represented) | ![Band-pass Transfer Function](image) |
| (c) High-pass | $T(s) = \frac{s^2}{s^2 + s \omega_0/Q + \omega_0^2}$  
  ($Q = 4$ represented) | ![High-pass Transfer Function](image) |
| (d) Band-stop | $T(s) = \frac{s^2 + s \omega_0/Q + \omega_0^2}{s^2 + s \omega_0/Q + \omega_0^2}$  
  ($Q = 4$ represented) | ![Band-stop Transfer Function](image) |
(e) All-pass

\[ T(s) = \frac{s^2 - s \omega_0/Q + \omega_0^2}{s^2 + s \omega_0/Q + \omega_0^2} \]

(Q = 4 represented)

Table 18.1-2. The equations shown are normalized to \(|T(\text{max})| = 1.0\) for BP, \(|T(0)| = 1.0\) for the LP and \(|T(\infty)| = 1.0\) for the HP. Biquadratic profiles serve as benchmarks for their higher-order cousins.

The profile for 18.1-2(e) (all-pass) is more correctly identified as a phase-shift circuit. The phase varies with frequency but the amplitude does not. Since it is second-order the maximum phase shift is \(-2 \times 90^\circ = -180^\circ\) as \(\omega \to \infty\).

Since the band-pass function is a profile that is supposed to select and pass a given frequency it is appropriate to identify the bandwidth of the profile \(\Delta \omega = \omega_2 - \omega_1\), for which \(\omega_1\) and \(\omega_2\) are the frequencies at the 3-dB level (i.e. \(|T| = 1/\sqrt{2}\)) as indicated by figure 18.1-3.

![Figure 18.1-3. (Normalized) biquadratic band-pass function](image)

Analysis of the normalized band-pass equation (given in Table 18.1-2(a) at magnitude levels for which \(|T| = 1/\sqrt{2}\), gives the quartic equation

\[ \omega = \pm \frac{\omega_0}{2Q} \pm \frac{\omega_0}{2Q} \sqrt{1+4Q^2} \]  

(18.1-5)

Of the 4 solution solutions the only ones that are greater than zero are

\[ \omega_1 = -\frac{\omega_0}{2Q} + \frac{\omega_0}{2Q} \sqrt{1+4Q^2} \quad \text{and} \quad \omega_2 = +\frac{\omega_0}{2Q} + \frac{\omega_0}{2Q} \sqrt{1+4Q^2} \]

for which \(\Delta \omega = \omega_2 - \omega_1 = \frac{\omega_0}{Q}\)  

(18.1-6a)
which represents the selectivity characteristic of the resonance peak, i.e.

\[ Q = \frac{\omega_0}{\Delta \omega} = \frac{f_0}{\Delta f} \]  \hspace{1cm} (18.1-6b)

which is why \( Q \) is called the ‘quality factor.’ The mathematics is detailed in Chapter 5 and equation (18.1-6b) is the same as equation (5.5-11). Otherwise emphasis should be made that \( Q \) is a magnitude factor for the low-pass profile resonance peak since

\[ \frac{|T(\omega = \omega_0)|}{|T(\omega = 0)|} = Q \]  \hspace{1cm} (18.1-7)

and which was examined more carefully in chapter 17 as well as in chapter 5.

As represented by figure 18.1-2 the low-pass and high-pass profiles of primary interest are (1) \( Q = 1/\sqrt{2} \), for which the profile is maximally flat and (2) \( Q > 4 \), for which the resonance peak begins to dominate.

![Figure 18.1-2: Low-pass profile for several values of \( Q \), i.e. \( Q = \{0.5, 0.707, 2, 4\} \). The resonance peak is approximately at \( f_0 \) for \( Q > 4 \).](image)

Specific applications of low-pass profiles are categorized according to their profile feature as follows:

1. Butterworth profiles = maximally flat in the pass-band
2. Chebyshev profiles = maximum roll off at the pass-band corner
3. Bessel (Thomson) = maximum linearity in the phase response and minimum overshoot for impulse signals

There are other profiles which accept ripple in both pass-band and stop-band in order to achieve enhancement of these profiles. For the biquadratic profile these categories may be acquired by appropriate choice of \( Q \), as follows:
(1) Butterworth: \( Q = \frac{1}{\sqrt{2}} \)
(2) Chebyshev: \( Q = 1.0 \) (3dB ripple)
(3) Bessel: \( Q = 0.5 \)

The Chebyshev profile is a little more artistic than suggested by these simple relationships, since it may be either of three types. Type (1) has ripple in the pass-band, type (2) has ripple in the stop-band and type (3) (a.k.a. elliptic or Cauer response) has ripple in both pass-band and stop-band. The Chebychev name is a consequence of the mathematics, for which the poles and zeros are defined by Chebyshev polynomials. In the s-plane its poles will pattern in the form of an ellipse.

The ripple for the Chebyshev profile may also be of different relative magnitudes. As might be expected for the biquadratic form, a \( Q > 1.0 \) is also a citation for more ripple in the pass band and the benefit of a greater roll-off at the break point. The circuit designer then takes up a role as an artist in mathematics and circuit electronics.

18.2 NORMALIZED FORMS and FREQUENCY RESCALING

Many circuit topologies have been analyzed, addressed, profiled and tabulated by generations of electrical engineers. Tabulated forms are standardized to a characteristic frequency \( \omega_0 = 1.0 \) r/s or a feature frequency \( \omega_f = 1.0 \) r/s. This reference is identified as the normalized form of the specific topology and is the set of start values to which rescaling techniques are applied.

Frequency rescaling is the analytical process of rescaling the components of a circuit topology to one that will yield an exact copy of the profile at another frequency. Frequency rescaling can also be used to transform the circuit into an equivalent topology with the same profile, in which case it is called a frequency transformation.

Circuit profiles are defined in terms of a set of zeros in the numerator \( N(s) \) terms and a set of zeros in the denominator \( D(s) \), (= poles because of their effect on the profile). Poles and zeros each identify with a time constant. If the time constants are rescaled into a new \( s' = j\omega' \) plane then the topology profile will be exactly the same if \( \omega_f \times \tau_1 = \omega_f \times \tau_2 \). For the topologies for which all of the frequency dependent components are capacitances then \( \tau_1 = R_1C_1 \) and \( \tau_2 = R_2C_2 \), and the \( s \) and \( s' \) planes will relate to one another by

\[
\omega_1R_1C_1 = \omega_2R_2C_2
\]  

(18.2-1)

where the notation relationship is \( s = j\omega \) and \( s' = j\omega' \).

So if the resistance remains unchanged, i.e. \( R_2 = R_1 \), then (18.2-1) reduces to
\[
\omega_2 C_2 = \omega_1 C_1 \quad \text{(18.2-2a)}
\]

and the profile is reflected from the \( s \) to the \( s' \) plane by a rescale of the capacitance, according to

\[
C_2 = \frac{\omega_1}{\omega_2} C_1 \quad \text{(18.2-2b)}
\]

For example if the \( s' \) domain is a rescale of 1000 \( s \), for which \( \omega_2 = 1000 \omega_1 \), the capacitances must be 1000 times smaller. A 1.0mF capacitance would become a 1.0μF capacitance.

If the capacitance remains unchanged, i.e. \( C_2 = C_1 \), and the resistances are changed instead then

\[
\omega_2 R_2 = \omega_1 R_1 \quad \text{(18.2-3a)}
\]

and the profile is reflected from the \( s \) to the \( s' \) plane according to

\[
R_2 = \frac{\omega_1}{\omega_2} R_1 \quad \text{(18.2-3b)}
\]

For example if the \( s' \) domain is 1000\( s \) the resistances must then be 1000 times smaller. Usually a reduction in the size of the resistances is not practical, particularly if \( R_1 \) is in single digit ohms.

The more appropriate option is to change both \( R \) and \( C \) in a process identified as RC rescaling which is accomplished by a two-step procedure. (1) keep \( R \) unchanged, i.e. \( R_2 = R_1 \), and rescale the capacitances to a new frequency \( \omega_M \), i.e. \( \omega_M C_2 = \omega_1 C_1 \)

for which

\[
\omega_M = \frac{C_1}{C_2} \omega_1 \quad \text{(18.2-4a)}
\]

and for which

\[
\omega_M R_1 C_2 = \omega_1 R_1 C_1
\]

For step (2) keep \( C_2 \) constant and rescale the resistances, for which

\[
\omega_M R_1 C_2 = \omega_2 R_2 C_2 \quad \text{and} \quad R_3 = \frac{\omega_M}{\omega_2} R_1 \quad \text{(18.2-4b)}
\]

Now both \( R \) and \( C \) are changed and the profile is translated into the \( s' \) plane to an identical profile, usually at a higher frequency if the starting frequency is the normalized frequency \( \omega_1 = 1.0 \) rad/see

The process can be done in a different order as indicated by figure 18.2-1. The most likely option is to elect a final capacitance value \( C_2 \), let it define \( \omega_M \), and then let the resistance follow. Equations (18.2-4a) and (18.2-4b) should then be packaged as the RC rescaling algorithm:
The frequency \( \omega_M \) in equations (18.2-5) is defined as the rescaling frequency. The \( \omega_{\text{init}} \) and \( \omega_{\text{final}} \) are initial and final frequencies, respectively. The mathematics is further simplified if the initial frequency \( \omega_{\text{init}} = 1.0 \, \text{r/s}, \) i.e. that associated with a normalized profile.

The same process can be accomplished with inductances except by means of the time constant \( \tau = L / R \) for which the equivalent to equation (18.2-1) is then

\[
\omega L_1 / R_1 = \omega L_2 / R_2
\]

(18.2-6)

but it is not common to rescale according to inductances.

A diagrammatic representation of \( RC \) rescaling using equations (18.2-5) is shown by figure 18.2-1.

\[
\omega_M = \frac{C_{\text{init}}}{C_{\text{final}}} \omega_{\text{init}} \quad (18.2-5a)
\]

and

\[
R_{\text{final}} = \frac{\omega_M}{\omega_{\text{final}}} R_{\text{init}} \quad (18.2-5b)
\]

For topologies that include both capacitances and inductances the rescaling process is not quite as simple. For example, the \( RLC \) biquadratic circuits will have

\[
\omega_0^2 = \frac{1}{LC} = \frac{R}{L} \times \frac{1}{RC} = \frac{1}{\tau_L} \times \frac{1}{\tau_C}
\]

(18.2-8)
So the resistances cannot be rescaled by the simple time-constant method applied to \( L \) and \( C \) separately. A different path must be followed which includes an **impedance magnitude rescaling**. The rescaling algorithm will then be of the form

\[
R_2 = k_M R_i \quad \text{(18.2-9a)}
\]

\[
C_2 = C_i \left( k_M k_f \right) \quad \text{(18.2-9b)}
\]

\[
L_2 = L_i \left( k_M k_f \right) \quad \text{(18.2-9c)}
\]

Quadratic rescaling must then be accomplished by means of either (1) a component translation which retains the simplicity of equations (18.2-1) or (18.2-7) or (2) paths like those represented by equations (18.2-9). Since the translation is essentially an equivalent circuit form it invites usage of amplifier-driven components and the reconfiguration of the topologies into an **active-filter** form.

### 18.3 BIQUADRATIC ACTIVE FILTER TOPOLOGIES

There are a number of \( RC \) topologies that can be invited to the biquadratic table. Almost all of them employ one or more gain elements, typically of the form of an opamp. The ideal opamp is usually assumed. Proof tests using (the spice) circuit simulation utility then replace the ideal opamp with opamp models from the parts library. The complete menu of biquad profiles are given in table 18.1-2.

The **Sallen-Key** circuit topology shown by figure 18.3-1 is a benchmark standard. It is simple, requires only one amplifier component, and the \( Q \) of the circuit is separately tunable from the characteristic frequency \( f_0 \).

![Figure 18.3-1 Sallen-Key single-amplifier biquad.](image)

Nodal analysis at \( v_1 \) and \( v_+ \) gives

\[
v_1 (G_1 + G_2 + sC_1) - v_+ sC_1 - v_+ G_1 - v_+ G_2 = 0
\]

\[
v_+ (G_2 + sC_2) - v_1 G_2 = 0
\]
Using $v_o = Kv$, where $K = 1 + R_B/R_A$ gives

$$T(s) = \frac{v_o}{v_s} = \frac{K}{s^2 + s\frac{G_2}{C_2} + \frac{G_1 + G_2}{C_1} + \frac{G_1 G_2}{C_1 C_2}}$$

Typically the capacitances are chosen as $C_1 = C_2 = C$ and conductances $G_1 = G_2 = G$ for which

$$T(s) = \frac{v_o}{v_s} = \frac{K\omega_0^2}{s^2 + s\omega_0 (3-K) + \omega_0^2}$$

where $\omega_0 = G/C = 1/RC$. Equation (18.3-2) is of the low-pass form, with $Q = 1/(3-K)$. So $Q$ can be tuned by means of an adjustment of the ration of $R_B/R_A$ according to

$$Q = 1/(2 - R_B / R_A)$$

Since the ratio $G_2/C_2$ is consistent throughout equation (18.3-1) the Sallen-Key biquad can be tapered by selecting $C_2 = aC_1 = aC$ and $G_2 = aG_1 = aG$. This modification does not change the characteristic frequency $\omega_0$, but will change the form of the expression for quality factor to

$$Q = 1/(1 + a - R_B / R_A)$$

If the gain element is set up for $K = 2$ (i.e. $R_B/R_A = 1$) then equation (18.3-4) simplifies to $Q = 1/a$ and the tapering factor may be used to predefine $Q$ as shown by figure 18.3-2.

![Figure 18.3-2. Design-B of the Sallen-Key for tunable Q.](image)

Note that Figure 18.3-2 shows the design in normalized form ($\omega_0 = 1.0r/s$), which is the usual start for application of the $RC$ rescaling algorithm (18.2-5). Figure 18.3-3 shows a spice rendition for $f_0$ rescaled to 10kHz, $C_p = 1.0nF$ and the tapering factor $1/Q$ stepped by the parametric menu.
Figure 18.3-3. Design B of the Sallen-Key with RC-rescaling to 10kHz and $Q$ stepped through \{0.707, 2, 5\}. Also note that the zero-frequency gain $K = 2.0$.

Another option of the Sallen-Key topology is the Saraga design shown by figure 18.3-4.

Figure 18.3-4. Saraga low-sensitivity design for the Sallen-Key single-amplifier biquad with $\omega_0$ normalized to 1.0 r/s. The design is appropriate to circumstances in which the component values have relatively poor tolerances.

**Two Integrator loops:** Since opamps are packaged integrated circuits, any number can be used in a circuit, and sometimes it is of practical benefit to do so as represented by the two-integrator loop of figure 18.3-5. It has three output points, each of which differs by the factor $-\omega_0/s$.

Figure 18.3-5. Two-integrator loop.
The factor $-\omega_0/s$ is generated by the ‘Miller’ integrator

\[
- \frac{\omega_0}{s}
\]

which is a straightforward and simple circuit. The two-integrator loop consists of two Miller integrators and one inverter-summing circuit as shown by figure 18.3-5, for which the output of the summing circuit will be

\[
v_o = \frac{1}{Q} \left( -\omega_0 \right) v_i - \frac{\omega_0^2}{s^2} v_o + n_2 v_i
\]  
\[(18.3-5)\]

Collecting like terms and resolving the transfer function from $v_i$ to $v_o$ yields high-pass transfer function

\[
\frac{v_o}{v_i} = \frac{n_2 s^2}{s^2 + s \omega_0 / Q + \omega_0^2} = T_{HP}(s)
\]  
\[(18.3-6)\]

And the others evolve by the multiplication factor $-\omega_0/s$ between stages

\[
\frac{v_1}{v_i} = \frac{v_2}{v_i} \times \left( -\frac{\omega_0}{s} \right) = -\frac{n_2 \omega_0 s}{s^2 + s \omega_0 / Q + \omega_0^2} = T_{BP}(s) \quad \text{(18.3-7a)} \quad \text{band-pass}
\]

\[
\frac{v_2}{v_i} = \frac{v_1}{v_i} \times \left( -\frac{\omega_0}{s} \right) = \frac{n_2 \omega_0^2}{s^2 + s \omega_0 / Q + \omega_0^2} = T_{LP}(s) \quad \text{(18.3-7b)} \quad \text{low-pass}
\]

Two-integrator loops are also called state-variable filters since they can provide the three basic pass-band functions of low-pass, high-pass, and band-pass.

The two-integrator loop most often invoked is the Tow-Thomas, or ring-of-three topology shown by figure 18.3-6. One of the integrators is a lossy integrator, and it is this option that gives the circuit its operational benefit.
Nodal analysis at the (feedback) input of opamp U1 gives

\[ 0 = v_s G_3 + v_2 G_4 + v_1 (G_1 + s C_1) \]  \hspace{1cm} (18.3-8a)

and since U3 is a unity-gain inverter and U2 is a Miller integrator then

\[ v_2 = \frac{G_2}{s C_2} v_1 \]  \hspace{1cm} (18.3-8b)

Solving for \( v_1 \) then gives the transfer ratio

\[ \frac{v_1}{v_s} = \frac{-s G_3 / C_1}{s^2 + s G_1 / C_1 + G_2 G_4 / C_1 C_2} = T_{BP}(s) \]  \hspace{1cm} (18.3-9) \hspace{1cm} \text{band-pass}

and from equation (18.3-8b)

\[ \frac{v_2}{v_s} = \frac{-G_2 G_3 / C_1 C_2}{s^2 + s G_1 / C_1 + G_2 G_4 / C_1 C_2} = T_{LP}(s) \]  \hspace{1cm} (18.3-10) \hspace{1cm} \text{low-pass}

Typically it is convenient to (1) choose \( C_1 = C_2 = C \) and \( R_2 = R_4 = R \), for which \( \omega_0 = 1 / RC \). Then (2) choose \( Q = R_1/R \). Then (3) choose amplitude \( k = R_1/R_3 \). Using this tuning algorithm, the profile parameters \( \omega_0, Q, \) and \( k \) can be separately adjusted in the order (1) thru (3).

The tuning simplicity of the Tow-Thomas topology is even more evident when it is put in normalized form, for which we would let \( C_1 = C_2 = 1.0F \) and \( R_2 = R_4 = 1.0Q \). Then \( R_1 = Q \) and \( R_3 = Q/k \), and the
topology is of the form shown by figure 18.3-7, which also includes an extra summing amplifier for the generation of the band-stop function.

![Tow-Thomas topology](image)

**Figure 18.3-7.** Tow-Thomas topology in normalized form configured with extra summing amplifier U4 for the generation of band-stop function.

In normalized form equations (18.3-9) and (18.3-10) will be

\[
\frac{v_1}{v_s} = \frac{-sk/Q}{s^2 + s/Q + 1} = T_{BP}(s) \quad \text{(18.3-11a) band-pass}
\]

\[
\frac{v_2}{v_s} = \frac{-k/Q}{s^2 + s/Q + 1} = T_{LP}(s) \quad \text{(18.3-11b) low-pass}
\]

The band-stop output is at the output \(v_4\) of summing opamp U4 for which \(v_4 = v_s + v_2\) and gives

\[
\frac{v_4}{v_s} = \frac{v_s + v_2}{v_s} = 1 + \frac{-sk/Q}{s^2 + s/Q + 1} = \frac{s^2 + s(1-k)/Q + 1}{s^2 + s/Q + 1} \quad \text{(18.3-12)}
\]

Equation 18.3-12 is a band-stop function. Its simulation rendition for \(k = 0.4\) and \(Q = 4\) is shown by figure 18.3-8. In the special case for which \(k = 1\), the band-stop function becomes a band-reject form that blocks the transfer of any signal at \(f_0\).
Figure 18.3-8. Tow-Thomas simulation using tuning algorithm and normalized form as rescaled to 1.0kHz. Low-pass and band-stop outputs are shown. Peak amplitude of the low-pass is 0.4 and the minimum value of the band-stop is 0.6. Note that the Tow-Thomas topology acts as a filter and not as a resonator (i.e. no outputs have greater magnitude than $v_i$).

If $k$ is set $= 2$ in equation (18.3-12) then a special case biquadratic function occurs at $v_4$ with transfer function

$$T(s) = \frac{s^2 - s/Q + 1}{s^2 + s/Q + 1}$$

(18.3-13)

This function is the all-pass function. It serves as a means to produce a phase shift as a function of frequency, centered at -90° at the (characteristic) frequency $f_0$.

### 18.4 Biquadratic Filter Functions and Higher-Order Profiles

The biquadratic denominator $D(s) = s^2 + s\omega_0/Q + \omega_0^2$ will have roots in the s-plane of

$$s = -\frac{\omega_0}{2Q} \left(1 - j\sqrt{4Q^2 - 1}\right) \quad \text{and} \quad s = -\frac{\omega_0}{2Q} \left(1 + j\sqrt{4Q^2 - 1}\right)$$

(18.4-1)

The roots are complex when $Q > 0.5$.  

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The s-plane is the domain used for complex frequency response. The denoted roots are located in the left (negative) half of the s-plane. As parameters $\omega_0$ and $Q$ are varied the biquadratic roots will follow a root-locus plot of these roots, as represented by figure 18.4-1.

![Figure 18.4-1. Root-locus plot for the poles of the biquadratic function. This figure is a modified copy of Figure 17.5-1 with more emphasis on the role of $Q$.](image)

The root-locus plot for $\omega_0 = $ constant is a circle with radius $\omega_0$. Evaluating the magnitude of either of the roots of equation 18.4-1 will result in

$$|s| = \frac{\omega_0}{2Q} \sqrt{1 + (4Q^2 - 1)} = \omega_0$$

Equation (18.4-2)

For a circle of constant $\omega_0$ in the s-plane $s = \omega_0(\cos \theta + j \sin \theta)$. Equation (18.4-1) then identifies the relationship between $\theta$ and $Q$ as $\cos \theta = 1/2Q$

for which the choice of $Q = 0.707$ gives $\theta = 45^\circ$ as shown by figure 18.4-1(b).

Roots and root loci specifically identify a frequency profile by the location of its poles and zeros. For example, the low-pass Butterworth profile with a roll-off of $n \times (-20\text{dB/decade})$ has the form

$$|H(s)|^2 = \left| \frac{k}{1 + (s/j\omega_0)^{2n}} \right|$$

with roots at $s = \omega_0 e^{j(\pi + 2k\pi)/2n} = \omega_0 e^{jk\phi_k}$ and $k = 0, 1, 2 \ldots n$. (18.4-3b)

If $n = 2$, then the two roots that fall in the left half plane are at $\phi_k = 3\pi/4$ and $5\pi/4$ both of which identify with left-hand angles of $\theta = \pm 45^\circ$ and $Q = 1/\sqrt{2}$. This outcome confirms that the maximally flat profile is the Butterworth profile by another name. If $n = 4$, then the four roots that fall in the left half plane are
at $\phi_k = 5\pi/8, 7\pi/8, 9\pi/8, 11\pi/8$. These correspond to two values of $Q = 1/[2\cos(22.5^\circ)] = 0.541$, and $Q = 1/[2\cos(67.5^\circ)] = 1.307$, and can be accomplished by a couple of Sallen-Key topologies in cascade, as shown by figure 18.4-2.

The root-locus plots are also of advantage to the use special mathematical functions to identify pole and zero distributions that will effect a sharper cut-off at the break frequency $f_c$ of the profile. A fairly popular class of sharp cutoff functions is based on Chebyshev polynomials according to transfer function

$$H(s) = \frac{k}{1 + \varepsilon^2 \cos^2(C_n(\omega / \omega_c))^2}$$

(18.4-4)

where the function $C_n(x) = n(\cos^{-1} \omega / \omega_c)$ is a Chebyshev polynomial, and is probably familiar only to them who is well-versed in special mathematical functions. The factor $\varepsilon$ is called the ripple factor and defines the ripple in the pass-band. As represented by figure 18.4-3, which shows a 3dB ripple, the ripple factor $\varepsilon = 1$. Cutoff frequency is referenced to $f_c = 2\pi \times \omega_c$.

Figure 18.4-2. Use of Sallen-key topologies in cascade to generate 4th-order (maximally-flat) Butterworth response. Take note that the roll-off slope is –80dB/decade = same as 4× (–20dB/dec).

Figure 18.4-3. 4th-order Chebyshev profile with $\varepsilon = 1$. 
The choice of $\omega_01 = 0.951 \omega_c$, $Q_1 = 5.59$ and $\omega_02 = 0.443 \omega_c$, $Q_2 = 1.075$ for two Sallen-Key biquads in series will result in a -3dB Chebyshev 4th-order profile, as shown by figure 18.4-4.

Figure 18.4-3. Sallen-Key implementation of 4th-order Chebyshev profile with $\varepsilon = 1$. The accompanying trace is that for the 4th-order Butterworth profile.

A source of biquadratic topologies with names and provenance is located under the Texas Instruments Split-supply Analog Filter expert. It not only offers topologies for the standard biquadratic forms but deploys them according to the more common filter properties (i.e. Butterworth, Bessel and Chebyshev).

18.5 DOUBLY-TERMINATED RLC LADDERS and TRANSFORMATIONS

The section on biquadratic forms is a small slice of the collection of frequency profiles that are standardized. For applications and realizations beyond the biquadratic there are two means by which we may glean higher-order profiles: (1) poles and zeros defined by polynomials (e.g. Butterworth, Chebyshev, Bessel) and (2) RLC tables. The polynomials are probably more mathematically satisfying but the tables are more convenient. It is not uncommon to define the rescaling and transformation algorithms in terms of a doubly-terminated RLC topology, such as is represented by figure 18.5-1. A subset of the tables are shown by figure 18.5-2, and credit for them belongs to one of signature resources for analog filter design (M.E. Van Valkenburgh, Analog Filter Design, HRW, 1982).

Figure 18.5-1. Doubly-terminated RLC ladder topologies (equivalent forms)
### Doubly-terminated RLC ladder values for Normalized Butterworth

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### Doubly-terminated RLC ladder values for Normalized Chebyshev

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### Doubly-terminated RLC ladder values for Normalized Bessel

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C1 L2 C3 L4 C5 L6 C7 L8 C9 L10
Figure 18.5-2. Doubly-terminated RLC ladder tables (this is only a subset). The strip index at the top beginning with \( L_1 \) is associated with the RLC ladder above the tables and the strip index at the bottom beginning with \( C_1 \) is associated with the RLC ladder below the tables.

Direct simulations using the RLC ladder topologies and table values will yield a normalized low-pass profile. The categories (polynomial types) are listed above the tables. All of the values are in Ohms, Farads, or Henrys, depending on the component. The rest of the story is that the tables are the registration benchmark for a profile of any of the state variables, low-pass, high-pass, band-pass, band-stop. Profiles other than low-pass are realized by means of transformations from the low-pass to the profile of interest.

The mathematics of making a transformation is undertaken with the association of a transfer function \( T(s) \) being defined by impedance/admittance ratios and that if they are transformed in the same way, the ratio will remain the same. Add translation to this context and a low-pass can be translated into an equivalent form with two roll-off corners instead of one if the low-pass is regarded as a one corner profile referenced to zero. All that is then needed is to shift the zero to the right and the other corner emerges.

But the greater benefit of the process is that an RLC circuit can be translated into an equivalent form that makes use of impedance-converter circuits with opamps to eliminate inductances. The outcome is then one that can be micro-miniaturized since a VLSI chip can contain many opamp cells.

The active-filter form with opamps is also of benefit because it resolves as an RC circuit. The RC circuit can then be translated to the domain of interest using the RC rescaling algorithm of equation (18.2-5).

The most straightforward transformation is the RLC:CRD transformation, so-named because \( R \) translates to a \( C \), \( L \) values translate to \( R \) and \( C \) translates to an impedance-conversion construct element (= \( D \)) called a frequency-dependent negative resistance (FDNR). The concept is illustrated by Figure 18.5-3.

Figure 18.5-3. RLC:CRD transformation

The technique is also called the Bruton transformation and is primarily a mathematical manipulation, as represented by equation (18.5-2)
Figure 18.5-3(a) has transfer function

\[ T(s) = \frac{v_2}{v_1} = \frac{1/sC}{R + sL + 1/sC} \tag{18.5-1} \]

If both numerator and denominator in (18.5-1) are multiplied by (1/s) then

\[ T(s) = \frac{v_2}{v_1} = \frac{1/s^2C}{R/s + L + 1/s^2C} = \frac{1/s^2D}{1/sC'' + R' + 1/s^2D} \tag{18.5-2} \]

The relabeling of RLC to C’R’D creates the circuit of the form of figure 18.5-3b. Since the unidentified component has frequency dependence \(s^2\), it has no imaginary part and therefore no phase shift. So it is of the form of a frequency dependent negative resistance (FDNR).

The component (= D) is an opamp construct, usually accomplished by the Fleige generalized impedance converter topology shown by figure 18.5-4.

---

**Figure 18.5-4.** (a) Generalized impedance converter (GIC) topology and (b) translation into an FDNR (= D component).

Analysis of figure 18.5-3(a) is a matter of nodal analysis at each of the three nodes labeled \(v_0\). They are all virtually connected because of the nullator action of the opamps. Eliminating \(v_1\) and \(v_2\) from the resulting three equations gives

\[ Y_1Y_3Y_5 + Y_2Y_4Y_6 = 0 \tag{18.5-3a} \]

Which can be solved for \(Y_{in} = -Y_i\) as

\[ Y_{in} = \frac{Y_2Y_4Y_6}{Y_3Y_5} \tag{18.5-3b} \]
With the $R$ and $C$ choices identified by figure 18.5-3(b) and electing $R_3 = R_4$ yields the necessary behavior for the ‘D’ component as

$$Y_{in} = s^2 C_2 C_6 R_5 = s^2 D$$  \hspace{1cm} (18.5-4)$$

The component then has magnitude $D = C_2 C_6 R_5$ with behavior of that of an FDNR.

The mathematics outlined by this analysis is more easily accommodated if invoked by the parametric option of the simulation utility. The process is illustrated by figures 18.5-5 and 18.5-6:

**Figure 18.5-5(a)** Doubly-terminated RLC topology for a 5th-order Chebyshev with 1.0dB ripple in the pass-band. Note that the parameter values are the number values from Chebyshev (C) of figure 18.5-2.

**Figure 18.5-5(b)** Simulation results. Note that the breakpoint is at $f_0 = (1/2\pi) \times 1.0 \text{ rad/sec} = 160 \text{ mHz}$. So this profile is the normalized form.
Figure 18.5-6 Same as figure 18.5-5(a) except implemented in CRD form. The $L$’s are replaced by $R$’s, the $R$’s are replaced by $C$’s, and the capacitances are replaced by FDNR’s.

The parameterization in figure 18.5-6(a) is doing all of the mathematical grunt work, to include the frequency rescaling. At the very bottom of the figure it shows that the capacitance has been elected to be 0.01 μF. The rescaling algorithm according to equations (18.2-5) is located in the parameter list in the lower left corner. The 1.0/r/s normalized frequency is translated into 0.16 since the simulation is always in Hz. The units are omitted since it is the mathematics that carries the process along. The scaling factor shown as $kR$ is applied to all resistances, exactly as prescribed by equation (18.2-5(b)).

The diagram is a little busy, but take note of the simplifications used and which are not uncommon.

Due to the RLC:CRD transformation the capacitances relate to the $R = 1.0$ Ω terminations, and are therefore normalized to 1.0F. The other capacitances belong to the FDNR and are arbitrary, but might as well be elected to be the same, i.e. 1.0F capacitances. This option also simplifies the (FDNR) value for D, which by equation (18.5-4) gives

$$D = C_2 C_6 R_5 = 1.0 \times 1.0 \times R_5 = R_5$$  \hspace{1cm} (18.5-5)

So it is not uncommon to deploy the CRD representation in the form of an equal capacitance realization.

And that lets the rescaling be a relatively simple $RC$ rescaling process, as indicated by the parametric usage in the lower right corner of figure 18.5-5(a).

For the final frequency (labeled as $f_C$ in the figure), the rescaled CRD realization gives the frequency profile result as shown by figure 18.5-5(b).
Figure 18.5-5(b) Simulation results. Both normalized profile and rescaled profile at $f_C = 10$ kHz are shown as a comparison to confirm that the transformation/rescaling will realize the same profile but at a different frequency.

There are a few extra wrinkles that are necessary which the mathematics does not represent. The capacitances that replaced the terminations (at each end) leave the circuit floating, and therefore must be accompanied by leakage resistances (shown as $999M\Omega$ values). If this is not done the circuit simulator will abort and react with an error message to this effect. The values of the leakage resistances are arbitrary but large.

The RLC-CRD process is a generic and common choice in active-filter design. Others of note are ‘LeapFrog’ design, which use a clever process to transform the RLC ladder to an equivalent form that will accomplish to a ‘low-pass to band-pass’ (LP:BP) transformation. But in the interest of keeping the topic area of frequency profiling reasonably compact, LeapFrog design will be omitted except for name acknowledgement. Regrets.
PORTFOLIO and SUMMARY

RC frequency rescaling

\[ \omega_M = \frac{C_{init}}{C_{final}} \omega_{init} \quad \text{and} \quad R_{final} = \frac{\omega_M}{\omega_{final}} R_{init} \]

Sallen-Key biquad (lowpass) = tuneable \( Q \)

\[ \omega_0 = 1/RC \]

\[ Q = 1/(1 + a - R_f/R_x) \]

where \( a \) = tapering factor

Tow-Thomas biquad = biquad states for (1) low-pass, (2) band-pass, (3) band-stop, (4) all-pass

\[ \omega_0 = 1/RC = 1.0 \text{ r/s} \quad \text{normalized} \]

Normalized values:

\[ C_1 = C_2 = 1.0 F \]
\[ R_2 = R_4 = 1.0 \Omega \]
\[ R_1 = Q \]
\[ R_3 = Q/k \]

\[ T_{BP}(s) = \frac{v_1}{v_s} = \frac{-sk/Q}{s^2 + s/Q + 1} \]

\[ T_{LP}(s) = \frac{v_2}{v_s} = \frac{-k/Q}{s^2 + s/Q + 1} \]

\[ T_{BS}(s) = \frac{v_s + v_1}{v_s} = \frac{v_4}{v_s} = \frac{s^2 - k/Q + 1}{s^2 + s/Q + 1} \]

\[ T_{AP}(s) = T_{BS}(s) \quad \text{when} \ k = 1 \]
Butterworth profiles = maximally flat in the pass-band
Chebyshev profiles = maximum rolloff at the pass-band corner
Bessel (Thomson) = maximum linearity in the phase response, minimum overshoot for impulse signals, minimum delay of signals

**Simulations: 5th-order low-pass. Profiles and pulse response**

Chebyshev (1.0dB)

Butterworth

Bessel
Toolboxes

**Twin-T**

**Sallen-Key**

**Fleige**

**Akers-Mossberg**

**Delyannis-Friend**