CHAPTER 4. TRANSIENT ANALYSIS OF ENERGY STORAGE COMPONENTS

4.1 INTRODUCTION

A circuit that includes energy-storage components will have a time-dependent behavior in I and V as these components are charged and discharged. In section 3.5 it was emphasized that L and C may also be characterized as storing current and voltage, respectively. Consequently it is the interaction of these components with the rest of the circuit that then characterize its I(t) and V(t) behavior.

Transient analysis is the I-V response of a circuit to which switch and impulse actions are applied. Switch and impulses are the actions for which an abrupt transition takes place. Since energy storage components charge and discharge with respect to time the circuit network will react and relax relative to these changes according to the (relaxation) time constants associated with the L and C components.

The mathematics of the charge and discharge of energy storage elements are defined by equations (3.5-1) and (3.5-3). Since \( I(t) = V/R \) in equation (3.5-1) and \( V(t) = IR \) in equation (3.5-3) these two equations are of the same form, i.e.

\[
y = -\tau \frac{dy}{dt} \quad (4.1-1a)
\]

where \( y \) is either \( V(t) \) or \( I(t) \) and \( \tau \) is a time constant associated with the conduction path. Equation (4.1-1a) is more transparent if rewritten as

\[
\frac{dy}{y} = -\frac{dt}{\tau} \quad (4.1-1b)
\]

Of course this is exactly what came forth with equations (3.5-6b) and (3.5-8b). Equation (4.1-1b) is accommodating to straightforward mathematical analysis inasmuch as

\[
\int \frac{dy}{y} = -\frac{1}{\tau} \int dt \quad \rightarrow \quad \ln(y) = -\frac{t}{\tau}
\]

for which, solving for \( y(t) \) gives

\[
y(t) = Ce^{-t/\tau} \quad (4.1-2)
\]

The rest of the story requires a look at individual circuit topologies, (simplest first). And from section 3.4 it should be expected that the time constants \( \tau_C \) and \( \tau_L \) for C and L, respectively, will become the defining time characteristic of the I(t), V(t) circuit transient response.
4.2 SINGLE TIME-CONSTANT TRANSIENT ANALYSIS

**Capacitance:** The simplest option is one in which charge and discharge of a capacitance is toggled by a switch, as represented by figure 4.2-1.

![Figure 4.2-1. Charge/discharge of capacitance through a resistance.](image)

For which the flow of charge is then

\[ I = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} \quad (4.2-1) \]

Using equation analysis by KVL and equation (4.2-1) gives

\[ V_1 = IR + V(t) = C\frac{dV}{dt}R + V(t) \quad (4.2-2) \]

Equation (4.2-2) is a first-order linear equation in \( V(t) \) for which \( V \) is the voltage across the capacitance.

It is of benefit to rewrite equation (4.2-2) as

\[ V - V_1 = -RC\frac{dV}{dt} = -\tau\frac{dV}{dt} \quad (4.2-3) \]

for which \( \tau = RC \quad (4.2-4) \)

The symbol \( \tau \) reflects that this is of the form of a time constant. More than that, it is the characteristic time constant of the \( RC \) charge/discharge and might be subscripted as \( \tau = \tau_C \) as was recognized in the previous chapter by equation (3.4-6b).

Equation (4.2-3) can be reordered and simplified as

\[ \frac{dV}{V - V_1} = -\frac{dt}{RC} = -\frac{dt}{\tau} \quad \Rightarrow \quad \int_{V_1}^{V} \frac{dV}{V - V_1} = \int_{0}^{\tau} dt \]
for which

$$\ln \left( \frac{V - V_1}{V(0) - V_1} \right) = -\frac{t}{\tau}$$

This result can be inverted and expressed in terms of its voltage profile as

$$V - V_1 = [V'(0) - V_1]e^{-t/\tau}$$  \hspace{1cm} (4.2-5)

$V_1$ is the voltage across the capacitance after a long time has elapsed and therefore might otherwise denoted as $V(\infty)$. Consequently equation (4.2-5) takes on the limit form

$$V = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$$  \hspace{1cm} (4.2-6)

The outcome reflects that the transient response is little more than an exponential transition between the two limit states $V(0)$ and $V(\infty)$ with response characteristic defined by $\tau$.

Under a switch mode the limits $V(0)$ and $V(\infty)$ reflect the two toggle states. The transitions between them are exponential as represented by figures 4.2-2(a) and 4.2-2(b) and equations (4.2-7a) and (4.2-7b) respectively.

**Figure 4.2-2(a).** $RC$ network with input switch toggled to $V_1 = 0$. Then $V(t)=0$ and $V(\infty) = V_B$.

For figure 4.2-2(a) and the values of $V(\infty)$ and $V(0)$ reflected by the switch, equation (4.2-6) gives

$$V = V_B e^{-t/\tau}$$  \hspace{1cm} (4.2-7a)

Equation (4.2-7a) represents the exponential decay of voltage $V(t)$ across a charged capacitance to zero, as indicated by the figure.
For figure 4.2-2(b) and the values of $V(\infty)$ and $V(0)$, equation (4.2-6) gives

$$V = V_B \left(1 - e^{-t/\tau}\right)$$

which reflects an exponential rise to $V_B$ as indicated by the figure.

Switch actions also allow stored charges to redistribute, as represented by figure 4.4-3.

The total charge on the two capacitances before switch closure is

$$Q_{TOT} = Q_1 + Q_2 = C_1 V_1 + C_2 V_2$$

Charges move but no charge escapes when the switch closes. And so

$$Q_{TOT} = V_x C_{TOT} = V_x (C_1 + C_2)$$

Consequently the voltage $V_x$ that results after switch closure is

$$V_x = \frac{Q_{TOT}}{C_{TOT}} = \frac{C_1 V_1 + C_2 V_2}{(C_1 + C_2)}$$

Consider the following example:
EXAMPLE 4.2-1: Two capacitances within an IC (integrated circuit) are connected by a transistor switch. Determine (a) the equilibrium voltage that results after the switch is closed and (b) total stored energy before and after.

SOLUTION: (a) Using equation (4.2-8)

\[ V_x = \frac{C_1V_1 + C_2V_2}{(C_1 + C_2)} = \frac{10 \times 3 + 40 \times 0.25}{(10 + 40)} = 0.8V \]

(b) The energies before and after closing the switch are given by

\[ w(\text{before}) = w_1 + w_2 = \frac{1}{2} C_1V_1^2 + \frac{1}{2} C_2V_2^2 \]
\[ = \frac{1}{2} \times 10 \times 3^2 + \frac{1}{2} \times 40 \times 0.25^2 = 45fJ + 1.25fJ = 46.25fJ \]

\[ w(\text{after}) = \frac{1}{2} (C_1 + C_2)V_x^2 = \frac{1}{2} \times (10 + 40) \times 0.8^2 = 16.0fJ \]

(Where did the missing energy go?)

Example 4.2-1 should raise the question about the fact that energy (in the amount of approximately 30fJ) seems to have vanished!!? Since the components are ideal the question is valid. But in fact the transistor switch will have an internal resistance. And so the missing energy has dissipated in the switch resistance. Integrated circuits take advantage of this fact and can use an infra-red scanner to see which switches (transistors) light up during a logic sequence. This phenomenon then provides a means to debug the actual switch action that is taking place.

The rest of the story is that switches may do more than simply toggle a source ON/OFF. They may also act as a transfer connection between two distinct and separate RC circuits, each with a different time constants and with different boundary conditions. Consider the following example.

EXAMPLE 4.2-2: Solve for the quantities indicated and determine the analytical response for \( V_C(t) \).

SOLUTION: \( V(0) = 10V \) by inspection.
The time constant (not asked) that charges capacitance $C$ to $V_B$ is

$$\tau = 20k\Omega \times 50pF = 1.0\mu s$$

Voltage $V(\infty) = 2.0V$ (by inspection)

due to charge redistribution. i.e. $V(\infty) = \frac{10 \times 50 + 200 \times 0}{(50 + 200)}$

$$\tau_2 = 80k\Omega \times 40pF = 3.2\mu s$$ (**where the 40pF is due to 50pF in series with 200pF)**

The energies are:

- $w_C(\text{before}) = \frac{1}{2} C_i V_i^2 = \frac{1}{2} \times 50 \times 10^2 = 2.5nJ = w_{TOT}(\text{before})$
- $w_C(\text{after}) = \frac{1}{2} C_i V_z^2 = \frac{1}{2} \times 50 \times 2^2 = 0.1nJ$
- $w_2(\text{after}) = \frac{1}{2} C_z V_z^2 = \frac{1}{2} \times 200 \times 2^2 = 0.4nJ$

so $w_{TOT}(\text{after}) = 0.1nJ + 0.4nJ = 0.5nJ$ and this amount of energy is dissipated in the $80k\Omega$ resistance.

By equation (4.2-6) the analytical behavior after the switch is flipped is

$$V = V(\infty) + (V(0) - V(\infty)) e^{-t/\tau}$$

$$V = 2 + 8e^{-t/3.2\mu s}$$

**Inductance:** Whereas capacitances represent charge storage and the energy associated with stored charge, inductances represent flux storage and the energy associated with stored magnetic flux. Similar transient response result when an inductance is toggled by a switch. The distinction is that inductance reacts to a change of current through it by a responding with a reaction voltage (induced voltage) of

$$V = L \frac{dI}{dt}$$ (4.2-9)

This behavior is acknowledged by Faraday’s law of magnetic induction. Comparing equation (4.2-9) to equation (4.2-1) for the capacitance, it is evident that the components are alike but complementary. The polarity of the induced potential will act to oppose the change of current.

If applied to a circuit as represented by figure 4.2-4 similar mathematics to that of the charge/discharge of the capacitance will result.
Like the capacitance the circuit symbol for an inductor resembles its basic construct, i.e. loops of wire enclosing energy in the form of magnetic field. The analytical interpretation of the circuit of figure 4.2-4 may be analyzed by KVL as

\[ V_1 = IR + V_L(t) = IR + L \frac{dI}{dt} \]  \hspace{1cm} (4.2-10)

where the positive value for \( V_L \) is due to the fact that it will react to oppose the current through the resistance in accordance with Lenz’s law. Like that of the capacitance, equation (4.2-10) is a linear equation which can be rewritten as

\[ I - \frac{V_1}{R} = -L \frac{dI}{dt} = -\tau \frac{dI}{dt} \]  \hspace{1cm} (4.2-11)

where the time constant \( \tau \) is

\[ \tau = \frac{L}{R} \]  \hspace{1cm} (4.2-12)

Equation (4.2-11) is of the same form as equation (4.2-3) and so the solution will be of the same form except in terms of current instead of voltage, i.e.

\[ I - I_1 = [I(0) - I_1] e^{-t/\tau} \]  \hspace{1cm} (4.2-13)

where \( I_1 = V_1/R \) represents the current after a long time has elapsed. It is appropriate to adopt the same protocol as used for capacitance charge and discharge switching, i.e.

\[ I = I(\infty) + [I(0) - I(\infty)] e^{-t/\tau} \]  \hspace{1cm} (4.2-14)

And in like manner to the capacitance, toggling the switch in an inductance loop will give transient behavior for \( I(t) \) as defined by the energy charge and discharge of the inductance.
Equation (4.2-14) shows that the $RL$ transient response is entirely like its $RC$ cousin. For the two toggle options of the switch the current through the inductance as a function of time is indicated by figures 4.2-5(a) and 4.2-5(b) with corresponding analytical responses given by equations (4.2-15a) and (4.2-15b).

So according to equation (4.2-14) and the limit conditions identified by figure 4.2-5(a)

$$I = I(0)e^{-\frac{t}{\tau}}$$

(4.2-15a)

with exponential decay to zero as indicated by figure 4.2-5(a)

So according to equation (4.2-14) and the limit conditions identified by figure 4.2-5(b)

$$I = \frac{V_B}{R} \left(1 - e^{-\frac{t}{\tau}}\right)$$

(4.2-15b)

with exponential rise to $V_B/R$ as indicated by figure 4.2-5(b).

The storage of energy by an inductance is similar to that of the capacitance except that it stores current, held in a set of loops by magnetic field. For the voltage $V(t)$ induced across an inductance as it is charged by current $I(t)$ the energy that it stores will be an integral of power $p(t) = I(t)V(t)$ of the form
If the geometrical interpretation of an inductance represented by equations (3.3-6) and (3.3-7) are applied to equation (4.2-16) then energy density equation (3.1-2) will result.

4.3 TRANSIENT ANALYSIS OF RLC TOPOLOGIES

Inductance and a capacitance are complementary energy storage components. One stores voltage and the other stores current. When both are in a circuit the release of energy from one will be absorbed by the other. This action invokes an energy oscillation between the \( L \) and the \( C \), one that would hypothetically go on forever were it not for the presence of resistance, which is a dissipative component that dampens the oscillation.

RLC circuits are critical and essential to the entire spectrum of RF (radio-frequency) circuits

There are two basic RLC topologies to which all others RLC circuit forms will relate. They are represented by figures 4.3-1(a) and 4.3-1(b).

Circuit analysis of the basic RLC topologies is not unlike that for the transient analysis of the basic single RC and RL circuits. Consider a KVL assessment of the topology of figure 4.3-1(a), for which

\[
V_S = iR + L \frac{di}{dt} + V_C
\]  

(4.3-1)

The substitution \( i = C \frac{dV_C}{dt} \) changes equation (4.3-1) to

\[
V_S = \left( C \frac{dV_C}{dt} \right) R + L \frac{d}{dt} \left( C \frac{dV_C}{dt} \right) + V_C
\]

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or (reorganizing)

\[ V_s = RC \frac{dV_C}{dt} + LC \frac{d^2V_C}{dt^2} + V_C \]

This equation may be restated as a (simplified and conventional) mathematical form in \( V_C \):

\[ \frac{V_s}{LC} = \frac{d^2V_C}{dt^2} + \frac{R}{L} \frac{dV_C}{dt} + \frac{1}{LC} V_C \]  \hspace{1cm} (4.3-2)

Similar results occur for figure 4.3-1(b) by applying nodal analysis at \( V_P \), for which

\[ GV_s = GV_P + C \frac{dV_P}{dt} + \frac{1}{L} \int_{-\infty}^{t} V_P dt \]

Taking the time derivative of this equation gives

\[ G \frac{dV_s}{dt} = G \frac{dV_P}{dt} + C \frac{d^2V_P}{dt^2} + \frac{V_P}{L} \]

which simplifies to

\[ \frac{G}{C} \frac{dV_s}{dt} = \frac{d^2V_P}{dt^2} + \frac{G}{C} \frac{dV_P}{dt} + \frac{V_P}{LC} \]  \hspace{1cm} (4.3-3)

Equations (4.3-2) and (4.3-3) are second-order mathematical forms in \( t \). The left-hand side in each case is of the same mathematical form, usually identified as the \textit{homogenous} form, and might then be restated as

\[ \frac{d^2x}{dt^2} + \frac{2 \xi}{\tau} \frac{dx}{dt} + \frac{x}{\tau^2} = 0 \]  \hspace{1cm} (4.3-4)

This form restates the coefficients in terms of time and damping constants. Time constant \( \tau \) relates to time constants \( \tau_L \) and \( \tau_C \) for the \( L \) and \( C \) components, respectively as

\[ \tau^2 = LC = \frac{L}{R} \times RC \]

or

\[ \tau^2 = \tau_L \tau_C \]  \hspace{1cm} (4.3-5a)

As an additional convenience and extension into the frequency domain, \( \tau \) can be restated in terms of a (radian) frequency

\[ \omega_0 = \frac{1}{\tau} \]  \hspace{1cm} (4.3-5b)

which results in a more tractable and more definitive second-order equation of the form
\[
\frac{d^2 x}{dt^2} + 2\xi \omega_0 \frac{dx}{dt} + \omega_0^2 x = 0
\]

In the time-honored way of solving equations that include derivatives, a WAG (wild-eyed guess) is assumed as the solution and then it is proven to be good by substitution back into the equation. In this case the choice \(x(t) = Ae^{\omega t}\) is made, for which, taking derivatives, gives

\[
(s^2 + 2\xi \omega_0 s + \omega_0^2) e^{\omega t} = 0 \quad (4.3-6)
\]

The WAG is confirmed if \((\ ) = 0\), for which the quadratic equation coefficient will then have roots

\[
s = -\xi \omega_0 \pm j \omega_0 \sqrt{1 - \xi^2} = -\sigma \pm j \omega_d \quad (4.3-7)
\]

Equation (4.3-7) makes the assumption that \(\xi < 1\) is the more likely option, so the roots will almost always be complex.

The solutions given by equations (4.3-6) and (4.3-7) show why coefficients \(1.0, 2\xi \omega_0\) and \(\omega_0^2\) are invoked. They are tailored to the fact that the \(RLC\) circuit is a resonant circuit in the frequency domain and this choice of coefficients relate to its frequency character. The parameter \(\omega_0\) is called the characteristic radian frequency and \(\xi\) is called the normalized damping coefficient. Appropriate the fact that the \(RLC\) equation is a quadratic form, two parametric characteristics, \(\xi\) and \(\omega_0\), are resolved.

On the frequency scale the characteristic frequency is

\[f_0 = \frac{1}{2\pi} \omega_0\]

For the signal being a step response like that generated by switch action, then \(V_S\) is one of two possible levels, in which case the response will be of the form

\[
x(t) = V(t) = V_0 e^{-\sigma t} = V_0 e^{-(\sigma \pm j\omega_d)} = (V_0 e^{-\sigma}) e^{\pm j\omega_d} \quad (4.3-8)
\]

Equation (4.3-8) is of the form of an exponentially decreasing amplitude with an oscillation of (radian) frequency \(\omega_d\) and amplitude decay factor \(\sigma = 1/\tau_R\) for which \(\tau_R\) is the amplitude decay time constant due to the resistance in the circuit. This behavior is confirmed by a simulation rendition of figures 4.3-1(a) and (4.3-1(b) in response to a step impulse (same as a switch toggle). The results are shown by the figures (4.3-2(a)) and (4.3-2(b)).
Figure 4.3-2(a). RLC series topology with step impulse at input. The input is indicated by the dashed line. The output is riding on the input and shows a ringing effect.

Figure 4.3-2(b). RLC parallel topology with step impulse of same amplitude as that of series topology except reduced by factor of 4 to show ringing response.

The response of an RLC circuit is not unlike that of one of its acoustic cousins in which a tap on a bell will give a ringing response that declines in amplitude with time constant $\tau_R$.

If the amplitude decay time constant $\tau_R$ is related back to the two topology options then

$$\frac{2\xi}{\tau} = 2\sigma = \frac{2}{\tau_R} = \frac{R}{L} = \frac{1}{\tau_L}$$

for the series topology

$$\frac{2\xi}{\tau} = 2\sigma = \frac{2}{\tau_R} = \frac{G}{C} = \frac{1}{\tau_C}$$

for the parallel topology

Via these interpretations, the amplitude decay constant $\tau_R$ relates directly to embedded components and their time constants according to the series/parallel topology options, i.e.

Series RLC: $\tau_R = 2\tau_L$ (4.3-9a)

Parallel RLC: $\tau_R = 2\tau_C$ (4.3-9b)

And $\xi = \tau / \tau_R$ (for both) (4.3-10)

EXAMPLE 4.3-1: (a) Analyze the RLC series circuit shown and determine all of the relevant time constants ($\tau_L$, $\tau_C$, $\tau_R$). (b) Determine $\xi$, $\omega_L$, and $f_d = 0.16\omega_L$. (c) Execute the circuit in pspice and extract $\omega_L$, $\tau_R$ and $\xi$ and compare to part (b).

Figure E4.3-1(a): RLC series topology example.
SOLUTION: (a) \( R_L = L/R = 10\mu H/50 = 0.2\mu s = 200\text{ns} \)
\( R_C = RC = 160\text{ps} \times 50 = 8.0\text{ns} \)
\( \tau = \sqrt{R_L \tau_C} = \sqrt{200 \times 8.0} = 40\text{ns} \)
\( \tau_R = 2\tau_L = 2.0 \times 0.2\mu s = 0.4\mu s \)

(b) \( \xi = \frac{\tau}{2\tau_L} = \frac{40\text{ns}/(2 \times 200\text{ns})}{0.1} \)
\( \omega_0 = \frac{1}{\tau} = \frac{1}{40\text{ns}} = 25\text{Mr}/\text{s} \)
so \( \omega_d = \omega_0 \sqrt{1 - \xi^2} = 25 \sqrt{1 - 0.1^2} = 24.88\text{Mr}/\text{s} \)
\( f_d = 0.16 \times 24.88 = 3.98\text{MHz} \)

(c) The circuit rendering in pspice is shown by figure E4.3-1(b).

The cursor coordinates show the measure of the first two amplitude peaks, both of which are \( \Delta V \) riding on a 1.0 V pulse amplitude. So the ringing amplitudes are

\( \Delta V(t_1) = 1.7162 - 1.0 = 0.7162 \)
\( \Delta V(t_2) = 1.3861 - 1.0 = 0.3861 \)

And according to equation (4.38) \( \Delta V(t) = \Delta V(0) e^{-\sigma t} \)

with ratio between any two amplitudes \( \frac{\Delta V(t_2)}{\Delta V(t_1)} = \frac{\Delta V(0) e^{-\sigma t_2}}{\Delta V(0) e^{-\sigma t_1}} = e^{-\sigma (t_2 - t_1)} = e^{-\sigma \Delta t} \)

taking the inverse \( \ln \left( \frac{\Delta V(t_2)}{\Delta V(t_1)} \right) = -\sigma \times \Delta t \)

which gives \( \ln \left( \frac{0.3861}{0.7162} \right) = -0.618 = -\sigma \times 0.25\mu s = -0.25\mu s/\tau_R \)

or \( \tau_R = 0.25/0.618 = 0.40\mu s \)

Figure E4.3-1(b): pspice rendering of example circuit.
and since  \( \sigma = 1/\tau_R = \omega_0 \zeta \approx \omega_0 \zeta \)

then  \( \zeta \approx 1/(\tau_R \times \omega_0) = 1/(0.4 \times 24.8) \approx 0.1 \)

The extracted measures for \( f_d \) (and \( \omega_d \)) and for \( \zeta \) compare 1 to 1, relative to the usual limits of accuracy and the approximations made. The extracted information does not give \( \omega_0 \), so the approximation \( \omega_0 \approx \omega_d \) is necessary and will not survive with any accuracy if \( \zeta > 0.25 \).

The parameter \( \zeta \) is also called the normalized damping coefficient and plays a role in the ability to suppress the ringing effects represented by figure 4.3-3. This figure is a pspice rendition of a series RLC topology with impulse input and output taken across the capacitance.

**Figure 4.3-3**  RLC series topology with input step impulse at input and \( \zeta \) stepped through values of 0.1, 0.5, 0.707 and 1.0. When \( \zeta = 1.0 \) the ringing impulse is critically damped. The trace corresponding to critical damping \( \zeta = 1.0 \) is shown in bold. The pulse is shown as a dashed line.

The response of the RLC circuit relative to the normalized damping coefficient \( \zeta \) are:

\[
\begin{align*}
\zeta &< 1 & \text{underdamped (ringing occurs)} \\
\zeta &= 1.0 & \text{critical damping} \\
\zeta &> 1 & \text{overdamped}
\end{align*}
\]

Damping is of significant importance to the pulse trains associated with logic information.
PORTFOLIO and SUMMARY

Transients and switching:

Charge redistribution:
\[ V_x = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \]

Transient analysis:
\[ x(t) = x(\infty) + [x(0) - x(\infty)] e^{-\frac{t}{\tau}} \]
where \( x(t) \) = either \( V(t) \) or \( I(t) \)
and \( \tau = RC = \tau_C \) or \( \tau = \frac{L}{R} = \tau_L \)

Exponential decay from high to low

Exponential rise from low to high

RLC circuits with impulse: Damped harmonic ringing response (exponential decay of amplitude)

\[ V(t) = (V_0 e^{-\sigma}) e^{i \omega_0 t} \]
\[ \Delta V(t) = \Delta V(0) e^{-\sigma} \]
\[ \sigma = \frac{1}{\tau_R} = \omega \xi \approx \omega_0 \xi \]
\[ \omega_0 = \omega_0 \sqrt{1 - \xi^2} \]
where \( \omega_0 = 1/\tau \)
\[ \tau^2 = LC = \tau_L \tau_C \]
\[ \sigma = \frac{1}{\tau_R} \]

Series RLC: \( \tau_R = 2\tau_L \)
Parallel RLC: \( \tau_R = 2\tau_C \)

\( \tau_C = RC = C/G \)
\( \tau_L = L/R \)
$$\xi = \text{normalized damping coefficient} = \frac{\tau}{\tau_R} = \frac{1}{(\omega_0 \tau_R)}$$

$$\xi < 1 \text{ underdamped, } \quad \xi > 1 \text{ overdamped, } \quad \xi = 1 \text{ critically damped}$$

The enhanced trace is the case for which $\xi = 1$