CHAPTER 6. THE IDEAL OPAMP

6.0 SIGNALS AND AMPLIFIERS

Systems of all types, electronic and otherwise, identify with an input and an output. In the electronics realm the charter of a system is dedicated to the conditioning of signals, usually those with frequency and amplitude. The basic system has two ports, one of which is an input port and the other of which is an output port. Since one of the principal requirements of a signal-conditioning system is amplification of a small signal to one large enough to have a significant effect on a load, the largest category of two-port networks are amplifiers and follow the context reflected by figure 6.0-1 (same as figure 2.5-3).

Figure 6.0-1. Signal transfer: Basic two-port systems

As represented by figure (6.0-1(a) the transfer characteristics are defined by $R_{in} =$ input resistance, $R_{out} =$ output resistance, and voltage transfer gain $\mu = \frac{A_{v}}{V_{o}/V_{in}}$. If these quantities are constant the signal transfer is linear and the output is a carbon copy of the input signal, but with more strength and power.

The voltage gain and the voltage-divider transfer ratios of an amplifier define the signal strength at the load, as represented by figure 6.0-2 (same as figure 2.5-4) and by equation (6.0-1).

Figure 6.0-1. Transfer of a signal from source ($v_S, R_S$) to load ($v_L, R_L$) via two-port interface

$$\frac{v_L}{v_S} = \frac{v_{in}}{v_S} \times \frac{v_{out}}{v_{in}} = \frac{R_{in}}{R_S + R_{in}} \times \frac{R_L}{R_{out} + R_L} $$

(6.0-1)
6.1 THE OPERATIONAL AMPLIFIER

The operational amplifier is a multi-transistor circuit engineered to have high input resistance \( R_{in} \), low output resistance \( R_{out} \), and high (voltage) transfer gain \( A_V \). Equation (6.0-1) shows that the signal strength is strongest for an amplifier with high \( R_{in} \) and low \( R_{out} \).

High \( R_{in} \) allows the amplifier to sample voltage as if it were a probe, since very little current can flow into such an input. This type input is also called a ‘buffer’ input. Low \( R_{out} \) implies that it drive a load with virtually no qualification. The opamp (operational amplifier) is a differential amplifier, i.e. it has two inputs and \( v_{out} = A_V \times v_{DIFF} = A_V \times (v_+ - v_-) \). Since it is regarded as a component as much if not more than a circuit it has a dedicated amplifier symbol as indicated by figure 6.1-1(a) or 6.1-1(b).

![Figure 6.1-1. The operational amplifier (opamp).](image)

The real device includes power rails and is represented by figure 6.1-1(b). In addition to two-port characteristics \( R_{in} \), \( R_{out} \), and \( A_V \) being finite the real opamp has other performance constraints and qualifications, and these caveats will be considered in a later chapter.

Whether ideal or real, the opamp is a two-input differential (difference) amplifier with a single output \( v_{out} = A_V \times (v_+ - v_-) \). It looks like an arrowhead, which implies that its operation is unidirectional. For the ideal opamp the input resistance is infinite and therefore draws no current, not even so much as a fA. And since its output resistance = 0 it can provide any level of current, enough to start your car, wash your laundry, or displace a bothersome planet. ‘Ideal’ is a fairly generous accommodation, but is the best and simplest approach to use of this versatile component.

6.2 NULLATOR-NORATOR CONTEXT of THE IDEAL OPAMP

The large gain of the opamp is of no great benefit unless it is included in a feedback loop. Circuit feedback is an option in which the output is fed back to the input, which is usually that of the gain component. The amplification strength of the gain element pulls the input into alignment with the
feedback network. This condition is commonly identified as negative feedback since it is an applied corrective action.

The high gain $A_V$ of the opamp makes this wrap emphatic. The context is illustrated by figure 6.2-1. In this illustration an $(R_1, R_2)$ voltage divider feeds is used to feed back a fraction of $v_O$ to the $(v_-)$ input node. Since the negative input $(v_-)$ causes the output to pull back, then the action is corrective and emphatic. Feedback to the positive $(v_+)$ input would have the opposite effect and would push the output against one of the power rails, a less useful result.

![Figure 6.2-1. The opamp with feedback.](image)

The feedback factor is identified as $\beta$ and is self-evident in figure 6.2-1. The mathematics of the feedback will be that a signal $v_F$ is fed back to the $v_-$ input from $v_O$, of the form

$$v_F = \beta \times v_O \quad = \left( \frac{R_1}{R_1 + R_2} \right) \times v_O$$

(6.2-1a)

The feedback factor $\beta$ is of some importance. In this topology it is due to the $R_1, R_2$ voltage divider and is

$$\beta = \frac{R_1}{R_1 + R_2}$$

(6.2-1b)

The rest of the story is that $(v_S - v_F) = v_O/A_V$. And since $A_V = \infty$ for the ideal opamp, then $v_S - v_F = 0$ as if virtually connected.

Since the ideal opamp also has $R_{in} = \infty$ between inputs then no current will flow between the inputs either.

The property $(v_I = 0, i_I = 0)$ is defined as a ‘nullator’ property (http://en.wikipedia.org/wiki/nullator) and is unique to the opamp. The property is conceptually illustrated by figure 6.2-1(c) which is a topological reorientation of figure 5-2.1(a). It represents the virtual connection effected by the nullator property.
The collateral property is that the ideal component can supply any voltage at any current level. That is also a little untrue. Such output is defined as a ‘norator’ (http://en.wikipedia.org/wiki/norator).

The ideal opamp is therefore characterized as a ‘nullator-norator’ component. Simulation software takes great delight in this simplicity but will give a misleading and flawed result when the opamp feedback is incorrectly applied. The appendix at the end of the chapter gives an example of this simulation flaw.

<table>
<thead>
<tr>
<th><strong>Nullator input:</strong></th>
<th>$v_I \rightarrow 0$ and $i_I \rightarrow 0$ courtesy of $R_{in} \rightarrow \infty$ Also called a virtual connection</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Norator output:</strong></td>
<td>$v_O = $ anything and $i_O = $ anything courtesy of $R_{out} \rightarrow 0$</td>
</tr>
</tbody>
</table>

But otherwise equations (6.2-1a) and (6.2-1b) give

$$\frac{v_{out}}{v_S} = \frac{1}{\beta} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1} = 1 + \alpha$$

(6.2-2)

Notice that the opamp seems to have disappeared from the mathematics and the transfer gain $v_{out}/v_S$ depends only on resistances $R_1$ and $R_2$. This effect is also emphasized by figure 6.2-1(c).

Equation (6.2-2) is the acknowledgement that the high gain of the opamp is the reason that it is rendered invisible. Otherwise it has pulled the input node(s) into alignment with the $v_F$ node of the voltage divider by the corrective effect of the negative feedback. Since the $v_S$ and $v_{out}$ nodes are then tightly aligned to the $R_1, R_2$ voltage divider, the characteristics of the transfer function $v_{out}/v_S$ are that of the voltage-divider network as if inverted.

A similar configuration with the same feedback topology is shown by figure 6.2-2. It has the same factor $\beta$ except that the input is connected to the lower end of the ($R_1, R_2$) network though $R_1$.

![Figure 6.2-2](image)

**Figure 6.2-2.** The opamp with feedback, same topology as figure 6.2-1, different input configuration. This configuration is called the inverted configuration.

Nodal analysis at the $v_F$ feedback node gives
\[ v_p (G_1 + G_2) - v_o G_2 - v_S G_1 = 0 \]  

(6.2-3a)

And since \( v_F \) is virtually (nullator) connected to \((v_+)\), then \( v_F \) is ‘pinned’ to GND (= 0 ), and

\[- v_o G_2 - v_S G_1 = 0 \]  

(6.2-3b)

with transfer gain

\[ \frac{v_o}{v_x} = - \frac{G_1}{G_2} = \frac{R_2}{R_1} = - \alpha \]  

(6.2-4)

Figure 6.2-2(a) is called the simple inverting configuration for the opamp, with ideal gain given by equation (6.2-4). The previous topology, figure 6.2-1(a) is identified as the simple non-inverting configuration with ideal gain given by equation (6.2-2).

These two topology options are benchmarks for all opamp circuit topologies. For emphasis and convenience the transfer properties of these benchmark configurations are summarized by table 6.2-1.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( A_{NI} )</th>
<th>( A_I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-inverting configuration</td>
<td>( 1 + R_2/R_1 )</td>
<td>( 1 + \alpha )</td>
</tr>
<tr>
<td>Inverting configuration</td>
<td>( -R_2/R_1 )</td>
<td>( -\alpha )</td>
</tr>
</tbody>
</table>

Table 6.2-1. Summary of the transfer gain analysis of benchmark \( R_1, R_2 \) opamp configurations

These configurations are more often oriented in a left-right form as shown by figure 6.2-3. Bend a few wires in your mind and you should be able to confirm that they are the same as figures 6.2-1 and 6.2-2.

It should be evident that figure 6.2-3 and table 6.2-1 are consistent. The topology orientations of figure 6.2-3 are of benefit to more extended topology options.

The distinction between the two basic configurations is the input point. The one for which the input is applied to the \((v_+)\) input of the opamp (i.e. the non-inverting topology) draws no current since its \( R_{in} = \infty \), input. The other option (inverting topology) admits current from \( v_S \) since a voltage difference has to exist.
across $R_1$ since the nullator property pins $v_F$ to zero (since $v_- = v_+$ = 0). The current into the node is then shunted past the virtual ground and through $R_2$ to the output node, (which is negative relative to $v_- $).

The input characteristics for the two configurations should therefore be denoted as follows:

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$R_{in}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-inverting configuration (input at $v_-$)</td>
<td>$R_{in} = \infty$</td>
</tr>
<tr>
<td>Inverting configuration (input to $v$ thru $R_1$)</td>
<td>$R_{in} (= v_S/i_S) = R_1$</td>
</tr>
</tbody>
</table>

Table 6.2-2. Input resistances for the two $R_1, R_2$ (benchmark) opamp topologies.

6.3 BUFFERED TOPOLOGIES: THE NON-INVERTING CONFIGURATION

Another way of summarizing table 6.2-2 is to say that the non-inverting topology is a ‘buffer’ circuit (i.e. draws no current).

Consider the use of an R-2R ladder as a feedback topology, as represented by figure 6.3-1. Simplified by the nullator-norator context it even looks like a ladder. The input $v_S$ will be virtually connected to the bottom rung, as shown by the figure.

The binary nature of the ladder will render a power of two for each rung, as indicated by the figure. It should be evident from figure 6.3-1(b) that $v_1 = 2v_S$. After that node $v_2 = 2v_1$, node $v_3 = 2v_2$, etc. by the nodal properties of the R-2R (binary) ladder. Consequently

$$v_o = 2^k v_S$$  \hspace{1cm} (6.3-1) \hspace{1cm}

and qualifies this topology as a standard topology and a portfolio circuit.
Any and all non-inverting configurations are buffer topologies (i.e. for which $R_{in} = \infty$), as represented by figure 6.3-2(a). It also may be configured as a unity-gain buffer as shown by figure 6.3-2(b).

The feedback factor $\beta$ the unity-gain buffer = 1.0 and is as near to as perfect a buffer isolation between input and output as possible, other than an optical link.

The R-2R ladder cited in the first example also finds a particular use on the front end as a digital-analog converter (DAC). A DAC has a set of inputs of the same $V_{BIT}$ magnitude $V_A$, $V_B$, $V_C$, $V_D$. Applied to the R-2R ladder and buffered by a non-inverting opamp configuration (as shown by the figure) it will retain the binary relationship to the inputs set and rescale the output to any desired voltage level.

By the nature of the R-2R ladder, each input further away from the $v_S$ node is reduced by a factor of two. If we analyze the circuit by superposition, starting with all inputs except $V_D$ set to zero, then we find

$$v_S = \frac{1}{3} v_D$$

(6.3-2)
So if we recognize (by the nature of the ladder) that the other inputs are successively smaller by a factor of two as we step backwards from $V_D = V_3$, then

$$V_{out} = \frac{1}{3} V_{BIT} \left( 1 + \frac{R_2}{R_1} \right) \left( b_0 + \frac{b_1}{2} + \frac{b_2}{4} + \frac{b_3}{8} + \cdots \right) \quad (6.3-3)$$

where $b_k = 0$ or 1 (bit value) corresponding to the eponymous inputs and $V_{BIT} =$ bit amplitude.

The DAC (*digital-analog-converter*) of figure 6.3-3 can be of any string length. The ratio $R_2/R_1$ is usually adjusted so that $V_{out}$ will reflect a voltage level consistent with the selected bit string input.

**EXAMPLE 6.4-1:** Suppose we have bit amplitude = 1.0V and we desire $V_{out} = 10V$ when input byte = $\$A = %1010$,

then $10 = \frac{1}{3} \times (1 + 0 + \frac{1}{4} + 0) \times (1.0) \times (1 + \alpha)$ which gives $R_2/R_1 = \alpha = 2.3$.

### 6.4 CURRENT THROUGH THE NULLATOR NODE (INVERTING CONFIGURATION)

Since the inverting configuration input $v_S$ is at the end of $R_2$ and the current through $R_1$ defines the current into the feedback network, then the inverting configuration provides a means to measure current. The $v_F$ (same as $v_-$) node voltage is virtually pinned to $v_+$ by the nullator input. Since $v_+$ usually is GND then this node is pinned to zero, which makes the mathematical analysis of inverting circuit topologies somewhat more friendly.

This property is very useful for measurement of device characteristics, as illustrated by figure 6.4-1.

**Figure 6.4-1(a).** Device I-V measurements using a virtual ground. Since no current can flow into the virtual ground it all flows through $R_F$ and therefore $V_{out}$ defines the current as

$$I(\text{device}) = -V_{out}/R_F \quad (6.4-1)$$
For an oscilloscope trace the reference is always to ground (channel #1). The current through the device also has to be referenced to ground (channel #2). The probe traces cannot use the same ground because the ground diverts the current that needs to be measured. So we use the opamp in an inverting topology to make the input to channel-1 relative to a virtual ground.

Consequently the I-V characteristics of the device under test can be directly displayed on the X-Y setting of the Oscilloscope as illustrated by figure 6.4-1(b). The trace shown is that for a junction diode.

Figure 6.4-1(b). XY output trace for the figure 6.4-1(a) test circuit with a junction diode as the device under test. The device under test can be any component or circuit, e.g. diode, transistor, CMOS logic inverter, etc, for which the I-V characteristics (its electrical performance) need to be examined.

The inverting topology also has the benefit of accepting all currents that enter the \((v_1)\) node and forwarding them downstream to \(v_{out}\) through feedback resistance \(R_2\) or whatever network exists as feedback from \(v_{out}\). It is therefore also called a *summing* input, as illustrated by figure 6.4-2.

Figure 6.4-2. Summing point nature of the inverting topology.

Since \(i_F = i_1 + i_2 + i_3\) then
Circuits, Devices, Networks, and Microelectronics

\[ v_o = - \left( \frac{R_F}{R_1} v_1 + \frac{R_F}{R_2} v_2 + \frac{R_F}{R_3} v_3 \right) \]  \hspace{1cm} (6.4-2)

For convenience and emphasis the resistance from \( v_o \) to \( v_- \) is labeled as \( R_F \) instead of \( R_2 \), citing and emphasizing its role as the feedback resistance.

Since the current into the opamp = 0 it doesn’t matter what is connected to input \( v_+ \), since no current can flow to it from \( (v_-) \). And since the output resistance of the opamp = 0 it doesn’t matter what load is at the output. This factoid is illustrated by figure 6.4-3.

![Figure 6.4-3](image)

Figure 6.4-3. Note that for the inverting configuration \( R_{in} \) always = \( R_1 \) and \( R_{out} \) always = 0.

6.5 OPAMP CIRCUITS and EXTENDED FEEDBACK NETWORKS

The optimal starting point for the analysis of a circuit defined by feedback is the feedback insertion node \( v_F \). For the opamp the feedback point \( v_F \) is the inverting input node (\( v_- \)).

As an example consider an opamp topology with the feedback defined by a ‘Tee’ network as shown by figure 6.5-1.
Applying nodal analysis at $v_F (= v_-)$ for figure 6.5-1 gives

$$v_F (G_1 + G_2) - v_1 G_2 - v_S G_1 = 0$$  \hspace{1cm} (6.5-1a)$$

Due to the virtual connection between the $v_+$ and $v_-$ inputs $v_F = v_+ \cong v_- = 0$ and (6.5-1a) reduces to

$$-v_1 G_2 - v_S G_1 = 0$$

and therefore

$$v_1 = -\frac{G_1}{G_2} v_S = -\frac{R_3}{R_1} v_S$$  \hspace{1cm} (6.5-1b)$$

(which is also the same as equation (6.2-4)). If nodal analysis is applied to node $v_1$, then

$$v_1 (G_2 + G_3 + G_4) - v_F G_2 - v_O G_4 = 0$$  \hspace{1cm} (6.5-2a)$$

But, since $v_F \cong v_+ = 0$, then

$$v_1 (G_2 + G_3 + G_4) - v_O G_4 = 0$$  \hspace{1cm} (6.5-2b)$$

or

$$v_O = \frac{(G_2 + G_3 + G_4)}{G_4} v_1 = \frac{(G_2 + G_3 + G_4)}{G_4} \left(-\frac{R_2}{R_1}\right) v_S$$  \hspace{1cm} (6.5-3)$$

Notice that equation (6.5-3 is the same as equation (6.2-4) except for multiplying factor $\frac{(G_2 + G_3 + G_4)}{G_4}$.
For the same topology revised as a non-inverting configuration (figure 6.5-2), the analysis process is the same, but with different outcome.

![Non-inverting configuration with Tee feedback network.](image)

**Figure 6.5-2.** Non-inverting configuration with Tee feedback network.

Nodal analysis at $v_F (=v_\_)$ gives

$$v_F(G_1 + G_2) - v_1 G_2 = 0$$  \hspace{1cm} (6.5-4a)

for which the nullator (virtual connection) condition $v_\_ \equiv v_+ = v_S$ at the input gives

$$v_1 = \frac{(G_1 + G_2)}{G_2} v_S = \left(1 + \frac{R_2}{R_1}\right) v_S$$  \hspace{1cm} (6.5-4b)

If we now apply nodal analysis at $v_I$, then

$$v_I(G_2 + G_3 + G_4) - v_2 G_2 - v_o G_4 = 0$$  \hspace{1cm} (6.5-5a)

This equation is identical to equation (6.5-2a) except that $v_\_ \equiv v_+ = v_S$, (nullator effect) then

$$v_1(G_2 + G_3 + G_4) - v_3 G_2 - v_o G_4 = 0$$  \hspace{1cm} (6.5-5b)

Collecting terms we have a result

$$v_o = \left[\frac{(G_2 + G_3 + G_4)}{G_4}\times\left(1 + \frac{R_2}{R_1}\right) - \frac{G_2}{G_4}\right] v_S$$  \hspace{1cm} (6.5-6)

Warning: this result is not of the same multiplier factor form as equation (6.5-3).

Even though equations (6.5-3) and (6.5-6) could be added to a table of formulas it would be somewhat ridiculous. It is preferable to not devise a formula for each and every feedback topology. Note that it
takes very little effort to step through the nodal analysis process to find transfer gain and/or feedback factor $\beta$. (Both of interest and importance in the assessment of opamp circuits.)

Emphasis is that the analysis should always be initiated at the feedback insertion point $v_F$. And then continue with a nodal analysis march through the rest of the feedback network.

The analysis is even easier when the resistances have values and/or symmetry. Consider the T-network of figures 6.5-1 and 6.5-5 with all $R = 10k\Omega$ (≡ equal resistance network):

---

**EXAMPLE 6.5-1:** Figures 6.5-1 and 6.5-2 with all resistances equal:

**SOLUTION:** Nodal analysis at $v_F$ (either figure) gives

$$v_F(G + G) - v_1 G - v_S G = 0$$

And therefore for figure (6.5-1) $v_1 = -v_S$ since $v_F = v_- = v_+ = 0$ (by virtual connection)

Continuing to a nodal analysis at $v_1$,

$$v_1(G + G + G) - v_F G - v_O G = 0$$

for which $v_1(G + G + G) = v_O G$ since $v_F = v_- = v_+ = 0$ (by virtual connection)

Therefore $v_O = 3v_1 = -3v_S$

For figure 6.5-2 and nodal analysis at node $v_F$:

$$v_F(G + G) - v_1 G - 0 \times G = 0$$

And since $v_F \cong v_+ = v_S$ (by the nullator virtual connection). Then $v_1 = 2v_S$

Continuing to nodal analysis at $v_1$:

$$v_1(G + G + G) - v_F G - v_O G = 0$$

Since by nullator input $v_F = v_S$ then $v_1(G + G + G) - v_S G = v_O G$ since

And therefore $v_O = 3 \times 2v_S = v_S = +5v_S$

---

**Recipe:**

1. Assume that the inputs are ‘virtually’ connected (nullator-norator concept).
2. Execute nodal analysis at $v_F$
3. Continue with nodal analysis along the feedback network until $v_{out}$ is included.
6.6 DIFFERENTIAL AMPLIFIER CONFIGURATIONS WITH OPAMPS

A number of applications relate to the use of the opamp as a drive element in diffamp configurations. Since the opamp is itself a diffamp it is a natural choice and provides relatively simple circuit options.

The basic differential amplifier construct is shown by figure 6.6-1

![Figure 6.6-1. Simple differential circuit driven by opamp.](image)

It is evident that \( v_2 \) is the inverting input since it is path connected to the inverting opamp input (\( v_− \)). The non-inverting input is the one that is path connected to \( v_+ \). It is also ratioed by the \( R_3, R_4 \) voltage divider with result

\[
v_+ = \frac{R_4}{R_3 + R_4} v_1
\]  

(6.6-1)

By superposition of the two inputs, the circuit of figure 6.6-1 will result in output

\[
v_{out} = \left(1 + \frac{R_2}{R_1}\right) v_+ - \frac{R_2}{R_1} v_2 \\
= \left(1 + \frac{R_2}{R_1}\right) \frac{R_4}{R_3 + R_4} v_1 - \frac{R_2}{R_1} v_2
\]  

(6.6-2)

Equation (6.6-2) is not very useful unless it can be revised so that

\[
v_{out} = \alpha \times (v_1 - v_2)
\]
This can be accomplished for $\alpha = R_2/R_1$ and $R_3$ and $R_4$ cleverly chosen to be $R_3/R_4 = 1/\alpha$. When applied to equation (6.6-2) this gives

$$v_{out} = (1 + \alpha) \times \left( \frac{1}{1 + 1/\alpha} \right) \times v_1 - \alpha \times v_2 = \alpha \times (v_1 - v_2)$$

which is the result desired.

However for best functionality as a diffamp the inputs need to be buffered. Neither input of figure 6.6-1 ($v_1$ nor $v_2$) has high input resistance. The most effective buffering is accomplished by the configuration of figure 6.6-5 for which the buffer inputs also include additional gain control. This topology is a standard and usually is a packaged part designated and identified as an instrumentation amplifier (IA).

![Figure 6.6-5. Instrumentation Amplifier.](image_url)

The advantage of this construct is not only that both inputs are buffered (high input resistance), but also the fact that the transfer gain can be adjusted by means of a single resistance $R_4$ according to

$$v_{out} = \alpha \times \left( 1 + \frac{2R_3}{R_4} \right)(v_1 - v_2)$$

and is a representative example of the nodal analysis technique of section 5-3 as shown by the following example:

**EXAMPLE 6.6-1:** Nodal analysis of figure 6.6-5

**SOLUTION:** Nodal analysis at $v_-(U3)$ and $v_-(U5)$, respectively, gives:

$$v_-(U3)G_3 + G_4 - G_3v_2 - G_4v_-(U2) = 0$$
\( v_-(U/2)(G_3 + G_4) - G_3v'_1 - G_4v_-(U/3) = 0 \)

Since \( v_-(U/3) \equiv v_2 \) and \( v_-(U/2) \equiv v_1 \) (virtual connections), then these equations become

\[
\begin{align*}
  v_2(G_3 + G_4) - G_3v'_2 - G_4v_1 &= 0 \\
v_1(G_3 + G_4) - G_3v'_1 - G_4v_2 &= 0
\end{align*}
\]

and if the second is subtracted from the first, then:

\[
(v_2 - v_1)(G_3 + G_4) - G_3(v'_2 - v'_1) - G_4(v_1 - v_2) = 0
\]

so that

\[
(v'_2 - v'_1) = \left(1 + \frac{2G_4}{G_3}\right)(v_2 - v_1)
\]

and

\[
v_{\text{out}} = \alpha \times \left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1) \quad \text{QED}
\]

Typically, we choose \( \alpha = 0.5 \) or 1.0 (small) and identify the differential gain of the IA by means of the coefficient factor \( \left(1 + 2G_4/G_3\right) = \left(1 + 2R_3/R_4\right)\).

### 6.7 FREQUENCY DEPENDENT NETWORKS USING (ideal) OPAMPS

The ideal opamp finds a role in an extensive number of frequency related applications, most of which are realized as resistance-capacitance networks. With translation of resistance to impedance, the non-inverting and inverting configurations then take on the form

\[
\frac{V_O}{V_S} = 1 + \frac{Z_2}{Z_1} \quad \text{and} \quad \frac{V_O}{V_S} = -\frac{Z_2}{Z_1}
\]

The inverting topology usually finds the most calling in signal transfer circuits since the impedance ratio will translate directly to a ratio of zeros and poles defined by the impedances.

As an example, one of the simplest of these ratio topologies is represented by figure 6.7-1, for which the feedback resistance \( R_2 \) of the inverting configuration is replaced by a capacitance \( C_2 \).
The large-signal analysis shown by the figure draws on the fact that \( i_2 = i_1 \) courtesy of the nullator at \( v_F \). Therefore

\[
i_2 = \frac{dQ_2}{dt} = C_2 \frac{d}{dt}(V_+ - V_{\text{out}}) = -C_2 \frac{dV_{\text{out}}}{dt}
\]

since \( V_+ = 0 \) by the nullator connection. Therefore

\[
i_1 = \frac{(V_S - V_+)}{R_1} = -C_2 \frac{dV_{\text{out}}}{dt}
\]

Integrating equation (6.7-1) resolves \( V_{\text{out}}(t) \) as

\[
V_{\text{out}}(t = T) = -\frac{1}{R_1 C_2} \times \int_0^T V_S(t) dt = -\frac{1}{\tau_{12}} \int_0^\tau V_S(t) dt
\]

This topology is also called an ‘accumulator’. If \( V_S(t) \) is a pseudo-random input with information buried in the noise a level shift will show up as an ‘accumulated’ signal level. An example of pseudo-random signal form with information in the form of impulses is that of a Geiger counter (random radiation detector).

The small-signal form given by figure 6.7-1(a) is

\[
\frac{v_{\text{out}}}{v_S} = -\frac{G_i}{sC_2} = -\frac{1}{sR_1 C_2}
\]

is also of significance, since it gives an output that is in quadrature (phase shift of 90°) with the input. The amplitude will also roll off linearly (at –20dB/dec) with respect to frequency, and is characterized by the time constant \( \tau_{12} = 1/R_1 C_2 \).
A modified form of the Miller integrator, known as the lossy Miller integrator, is shown by figure 6.7-2.

Applying nodal analysis at the \((v_\text{s} = v_F)\) input and recognizing that the virtual connection puts this node virtually \(= 0\) we get

\[
v_\text{o} = -\frac{G_1}{G_2 + sC_2} v_\text{s} = -\frac{R_2}{R_1} \times \frac{1}{(1 + sR_2C_2)} \times v_\text{s}
\]

which is the form of a low-pass profile with magnitude response

\[
\left| \frac{v_\text{o}}{v_\text{s}} \right| = \frac{R_2}{R_1} \times \frac{1}{\sqrt{1 + \omega^2 / \omega_2^2}}
\]

where \(\omega_2 = 1/R_2C_2 = 1/\tau_{22}\) and with a phase shift \(\phi = 180 - \tan^{-1}(\omega / \omega_2)\).

The resistance \(R_2\) is usually elected to be large for which the frequency corner \(= \omega_2\) is expected to be relatively low. This is consistent with the context of a leakage (or loss) and overcomes a flaw of the ideal Miller integrator (figure 6.7-1b) in which a slight parasitic input offset (not uncommon to real opamps) will accumulate and push \(V_{\text{out}}\) against the voltage rails, which totally compromises its function.

Equally simple, but somewhat less useful, is the Miller differentiator, represented by figure 6.7-3.
Figure 6.7-3. *Miller differentiator.* The resistance $R_1$ of the inverting configuration is replaced by a capacitance.

It has response (resolved by nodal analysis) of the form

$$V_{out} = -R_2C_1 \times \frac{dV_S(t)}{dt} \quad (6.7-5a)$$

And like its counterpart given by figure 6.7-1, it has a small-signal response that is a little strange and a little interesting, for which

$$v_{out} = -sR_2C_1v_S \quad (6.7-5b)$$

which linearly increases with respect to frequency and is characterized by time constant $\tau = R_2C_1$.

We may also indulge in topologies which include more than one time constant, such as the bandpass construct shown by figure 6.7-4.

Figure 6.7-4. Simple bandpass topology using ideal opamp.

This circuit topology has transfer function (cross-reference to the figure)

$$\frac{v_{out}}{v_s} = \frac{Z_2}{Z_1} = \frac{1}{Z_1Y_2} = -\frac{1}{(R_1 + 1/sC_1)(G_2 + sC_2)}$$
Equation (6.7-7) shows that a lower roll-off corner will occur at $\omega_1 = 1/R_1C_1$ and an upper roll-off corner at $\omega_2 = 1/R_2C_2$ for the Bode magnitude plot.

A large number of frequency/phase profiles may be devised using one or more opamps as network drivers. The transfer function will then be defined in terms of the linear components that make up the network, usually resistances and capacitances. In general these bear more qualification than purely resistive networks, since the non-ideal opamp will add frequency characteristics of its own.

### 6.8 OPAMP TOPOLOGIES EMPLOYING the ANTI-CORRECTIVE FEEDBACK LOOP

Constructs with anti-corrective (= positive feedback) loops generally have no merit by themselves but they can produce some interesting benefit when applied in concert with the corrective (negative) feedback, provided that the corrective feedback predominates. Consider the circuit of figure 6.8-1, which employs both corrective and anti-corrective feedback.

![Figure 6.8-1.](image)

In this circuit, it is assumed that the negative feedback loop defined by resistances $R_1$ and $R_3$ will prevail over the one formed by $R_5$ and $R_4$, so that

$$v_{out} = \left( 1 + \frac{R_2}{R_1} \right) v_s = (1 + \alpha)v_s$$  \hspace{1cm} (6.8-1)

where $\alpha = R_2/R_1$.

Application of nodal analysis at $v_+$ yields

$$v_+ (G_3 + G_4 + G_5) - v_+ G_3 - v_{out} G_5 = 0$$  \hspace{1cm} (6.8-2)
Using equation (6.8-1) we have

\[ v_+ = \frac{G_3}{(G_3 + G_4 - \alpha G_5)} v_S \]  

(6.8-3)

for which we then obtain

\[ \frac{v_{out}}{v_S} = \frac{(1 + \alpha)}{(1 + G_4/G_3 - \alpha G_5/G_3)} \]  

(6.8-4)

Note that the effect of the anti-corrective feedback is to include a subtractive term in the denominator.

If we should choose values such that \( G_5/G_3 = 1/\alpha \) (same as \( R_5/R_3 = R_2/R_1 \)) and replace \( G_4 \) by a capacitance (see figure 6.8-5), then

\[ \frac{v_{out}}{v_S} = \frac{(1 + \alpha)}{sC_4/G_3} = \frac{(1 + \alpha)}{sR_3C_4} \]  

(6.8-5)

\[ \text{Figure 6.8-5. Non-inverting integrator (as consequence of positive feedback).} \]

which is similar to equation (6.7-2b) for the Miller integrator except of a non-inverting form.

This is not a good circuit because both positive and negative feedback paths are balanced by the choice \( R_5/R_3 = R_2/R_1 \). If slight differences in the resistances allow the positive (anti-corrective) to prevail, the output will pin itself to one of the voltage supply rails. A ‘lossy’ version can be created by adding a resistance \( R_4 \) in parallel with \( C_4 \) and make the circuit more stable.
PORTFOLIO and SUMMARY

**Name:** basic non-inverting  **Variant1:** = unity-gain buffer  **Variant2:** = potentiometer

Since no current can flow into the virtual ground it all flows through $R_F$ and therefore $V_{out}$ defines the current as

$$I_{(device)} = -\frac{V_{out}}{R_F}.$$
Name: Instrumentation amplifier (IA)

\[ v_O = \alpha \times \left(1 - \frac{2R_2}{R_1}\right) (v_2 - v_1) \]

where: \( \alpha = R_f/R_1 \)

\[ T = \frac{R_y/R_1}{\left(1 + \omega_1 s\right)\left(1 + \omega_2 s\right)} \]

where: \( \omega_1 = \frac{1}{(R_yC_1)} \), \( \omega_2 = \frac{1}{(R_yC_2)} \)

Name: Bandwidth select
**APPENDIX 6-1**: Simulation example of the ideal opamp as a nullator-norator.

Topology #1 (figure A5-1.1a) is the correct form for use of the opamp in a non-inverting situation.

![Figure A5-1.1(a) Inverting topology](image1)

![Figure A5-1.1(b) Simulation output](image2)

![Figure A5-2.2(a) Inverting topology](image3)

![Figure A5-1.2(b) Simulation output](image4)

The outputs are the same (!!) even though topology #2 (figure A5-2.1b) is an invalid feedback topology. Note the polarity of the inputs. If a ‘real’ opamp were used, topology #2 will fail.

The simulation software is treating the opamp as a nullator-norator. But it is valid only if the feedback is to the \((v^-)\) input. In this instance the simulation utility has a subtle flaw since it will treat the ideal opamp as always having a nullator input even when the feedback is not to the \((v^-)\) input.