Problem 1. (20 pts.) A copper wire dipole antenna ($\sigma_c = 5.8 \times 10^7$ S/m, wire radius = 1.2 mm, overall wire length = 108 mm) is operated at 250 MHz and radiates 10 mW. Determine
(a.) the peak antenna current.
(b.) the power dissipated in the antenna.
(c.) the power density at a distance of 10 m in the direction of maximum radiation.
(d.) the antenna radiation efficiency.

Problem 2. (20 pts) A transmitter delivers 200 mW to the input of a lossless 75 $\Omega$ transmission line when the transmission line is terminated by a matched load. Assuming the matched load is removed and replaced by a resonant antenna ($R_{rad} = 50 \Omega$, $R_{tun} = 1 \Omega$), determine
(a.) the reflection coefficient at the antenna input.
(b.) the standing wave ratio on the transmission line.
(c.) the power reflected by the mismatched transmission line termination.
(d.) the power radiated by the antenna.

Problem 3. (20 pts) An antenna in air radiates the following far field power density:

\[ P_{av} = \frac{2}{R^2} \cos^4 \theta \hat{R} \quad (W/ m^2) \]

Determine
(a.) the radiation intensity function.
(b.) the total radiated power.
(c.) the antenna directive gain.
(d.) the beam solid angle of the antenna.

Problem 4. (20 pts.) A 1.5 GHz transmit/receive system utilizes antennas aligned for maximum gain with $G_t = G_r = 3.8$ dB. The transmitter output power is 3 W and the receiver requires an $SNR$ of 20 dB ($B = 5$ MHz, $T = 750$ K). A cable loss of 2.4 dB is present on both the transmit and receive sides of the system. Determine
(a.) the effective area of the receive antenna.
(b.) the total received power when the transmit and receive antennas are separated by the maximum separation distance ($R_{max}$).
(c.) the value of $R_{max}$.
(d.) the link gain when $R = R_{max}$.

Problem 5. (20 pts.) A 200 kW monostatic radar operating at 16 GHz uses a common transmit/receive antenna with a gain of 34 dB. The radar tracks a target with a radar cross section of 2.8 m$^2$ at a range of 7 km. Determine
(a.) the magnitude of the incident electric field at the target.
(b.) the total power captured by the target.
(c.) the magnitude of the scattered electric field at the radar.
(d.) the total power received by the radar.
1. (a) $P_{\text{rad}} = \frac{c \lambda}{\hbar c} = \frac{3 \times 10^8}{260 \times 186} = 1.2 \text{mW}$, $P_{\text{rad}} = 0.09 \text{ W}$

2. (a) $P_{\text{rad}} = \frac{c}{\hbar c} = 1.60 \text{ W}$, $P_{\text{rad}} = \frac{1.60}{\hbar c} = \frac{1.60}{0.09} = 112 \text{ mW}$

(b) $P_{\text{loss}} = \frac{1}{2} P_{\text{loss}}^2$, $P_{\text{loss}} = \frac{1}{2} \sqrt{P_{\text{rad}}}$, $P_{\text{loss}} = 29.5 \text{ mW}$, $P_{\text{loss}} = 0.185 \text{ mW}$

(c) $P_{\text{ave}} = \frac{\lambda}{\lambda^2} \left( \frac{2 \pi}{\lambda} \right)^2 \theta = 90^\circ$, $P_{\text{ave}} = \frac{\lambda}{\lambda^2} \left( \frac{2 \pi}{\lambda} \right)^2 60 \pi \left( \frac{0.12}{4 (10)} \right)^2 = 12 \frac{\text{W}}{\text{m}^2}$

(d) $\lambda = \frac{\lambda}{\lambda^2} = \frac{160}{29.5 + 1600} = 0.09 \frac{\text{m}}{\text{m}}$

3. (a) $\lambda (\theta, \phi) = R \lambda_{\text{ave}} (\theta, \phi) = 2 \cos^4 \theta$

(b) $P_{\text{rad}} = \frac{1}{2} \sum_{\phi} U(\theta, \phi) \sin \theta d\theta d\phi = 2 \int_{0}^{\pi} \cos^4 \theta \sin \theta d\theta = 2 \int_{0}^{\pi} \cos^4 \theta \sin \theta d\theta = 2 \left( \frac{\pi}{5} \right) = \frac{2 \pi}{5}$

(c) $S_{\text{d}} (\theta, \phi) = 4 \pi \frac{U(\theta, \phi)}{R_{\text{rad}}}$

(d) $S_{\text{d}} (\theta, \phi) = \frac{A_{\text{d}}}{4 \pi}$

4. (a) $A_{\text{d}} = \frac{2 \pi}{\lambda^2} = \frac{2 \pi}{180 \times 106} = 0.02 \text{ m}$

(b) $P_{\text{min}} = \frac{\lambda}{4 \pi} \left( \frac{138 \times 15}{5 \times 10^4} \right) = 5.12 \text{ W}$

(c) $R_{\text{max}} = \frac{\frac{4 \pi}{4 \pi} \left( \frac{0.2}{0.04 \times 10^{-12}} \right)^2}{\frac{4 \pi}{4 \pi} \left( \frac{0.2}{0.04 \times 10^{-12}} \right)^2}$

$d = 16.72 \text{ km}$

5. (a) $\eta = \frac{P_{\text{rad}}}{P_{\text{out}}} = 5.18 \times 10^{-12} / 3 = 1.72 \times 10^{-12} (-117.6 \text{ dB})$

(b) $P_{\text{c}} = \sigma \rho_{\text{i}} = 2.8 \times 0.816 = 2.28 \text{ W}$

(c) $P_{\text{i}} = \frac{\sigma V^2}{4 \pi R^2} = 3.70 \frac{\text{W}}{\text{m}^2}$

(d) $P_{\text{r}} = \frac{\lambda}{4 \pi} \left( \frac{3 \times 10^4 / 16 \times 10^9}{4 \pi \left( 3 \times 10^{-9} \right)} \right)^2 = 2.60 \text{ kW}$