Problem 1 (25 pts.) Two infinitely long parallel wires in air are separated by 10 cm and carry 15 A in opposite directions \( I_1 = 15 \vec{x} \) at \( y = 5 \text{ cm}, \ z = 0 \) and \( I_2 = -15 \vec{x} \) at \( y = -5 \text{ cm}, \ z = 0 \).

(a.) Determine the vector force per unit length acting on the conductor carrying \( I_1 \).

(b.) Determine the vector magnetic field at the rectangular coordinate point \( P = (0, 8, 0) \text{ cm} \).

(c.) If, in addition to the two infinitely long wires, a circular loop of 1 cm radius lies in the \( x-y \) plane centered at the coordinate origin, determine the vector loop current which would produce a magnetic field of zero at the loop center.

Problem 2. (25 pts.) A 5-turn rectangular wire loop occupies the surface defined by \( 0 \leq x \leq 0.2 \text{ m} \) and \( 0 \leq y \leq 0.4 \text{ m} \) in the \( x-y \) plane. Determine the induced emf across the open-circuited ends of the coil for the following magnetic flux conditions.

(a.) \( \mathbf{B} = 2xy^2 \mathbf{\hat{z}} \) (mT)

(b.) \( \mathbf{B} = x \cos(3y) \sin(10^4 t) \mathbf{\hat{z}} \) (mT)

(c.) \( \mathbf{B} = 2x \cos(y) \sin(10^4 t) \mathbf{\hat{x}} + x \cos(3y) \sin(10^4 t) \mathbf{\hat{z}} \) (mT)

Problem 3. (25 pts.) A long cylindrical conductor (radius = \( a \)) lying along the \( z \)-axis carries a current density characterized by \( \mathbf{J} = J_0 (1 - r/a) \mathbf{\hat{z}} \) A/m\(^2\) where \( J_0 \) is a constant and \( r \) is the radial distance from the cylinder axis. Assuming the conductor lies in air, use Ampere's law to determine an expression for the magnetic field

(a.) inside the conductor (\( r < a \)).

(b.) outside the conductor (\( r > a \)).

Problem 4. (25 pts.) The insulator in a parallel plate capacitor (plate area = 16 cm\(^2\), plate separation = 20 mm) is characterized by \( \varepsilon = 4.5 \) and \( \sigma = 10^{-12} \text{ S/m} \). A voltage \( v(t) = 10 \cos(\omega t) \) volts is applied across the capacitor plates, where \( \omega = 2\pi \times 10^7 \text{ rad/s} \). Determine

(a.) the electric field \( \mathbf{E}(t) \) between the capacitor plates.

(b.) the conduction current density in the capacitor.

(c.) the displacement current density in the capacitor.

(d.) the capacitance \( C \) in the capacitor equivalent circuit.

(e.) the resistance \( R \) in the capacitor equivalent circuit.
1. (a) \[ F = \frac{\mathbf{u}_0 I_1 \mathbf{I}_2}{2\pi a} \mathbf{y} = \frac{\mathbf{u}_0 (15)^3}{2\pi (0.10)} \mathbf{y} = \left[ 0.45 \mathbf{y} \right]_{mN} \]

(b) \[ \mathbf{H} = \frac{I_1}{2\pi r} \mathbf{J} = \frac{15}{2\pi (0.02)} \mathbf{J} = \left[ 15 \mathbf{J} \right]_{A/m} \]

(c) \[ \mathbf{H}_{loop} = -[H_1 + H_2] = - \left[ \frac{15 \mathbf{J}}{2\pi (0.02)} \right] = 95.49 \mathbf{A} \]

2. \[ \mathbf{V} = -N \frac{d\mathbf{M}}{dt} = -5 \frac{d\mathbf{B}}{dt} \cdot ds^2 \]

3. Ampere's Law on a circular path of radius \( r \)

\[ \oint \mathbf{H} \cdot d\mathbf{l} = \oint \mathbf{H}_0 \cdot d\mathbf{l} = H_0 \oint d\mathbf{l} = H_0 (2\pi r) = I_{enc} \]

(a) For \( r < a \), \[ I_{enc} = \int \frac{d\mathbf{B}}{dt} \cdot ds^2 = \int_0^a \int_0^{2\pi} \left[ H_0 \mathbf{J}_0 (1-r)^2 \right] r \cdot dsd\mathbf{\theta} \]

(b) For \( r > a \), \[ I_{enc} = \int \frac{d\mathbf{B}}{dt} \cdot ds^2 = \int_0^a \int_0^{2\pi} \left[ \mathbf{J}_0 (2\pi r) \right] r \cdot dsd\mathbf{\theta} \]

4. (a) \[ E(x) = \frac{V(x)}{d} = 10 \cos \omega t / 0.02 = \frac{500 \cos (2\pi 10^7 t)}{V/m} \]

(b) \[ \mathbf{J}(x) = \mathbf{E}(x) = (10^{-12}) 500 \cos (2\pi 10^7 t) = 0.5 \cos (2\pi 10^7 t) \mathbf{A}/m^2 \]

(c) \[ \mathbf{J}_d(x) = \frac{d\mathbf{B}(x)}{dt} = \epsilon \frac{d\mathbf{E}(x)}{dt} = 4.5 \epsilon_0 (500)(2\pi 10^7) [\cos (2\pi 10^7 t)] \]

\[ = -1.25 \sin (2\pi 10^7 t) \mathbf{A}/m^2 \]

(d) \[ C = \frac{\epsilon_0 \mathbf{A}}{d} = (4.5 \epsilon_0) (16 \times 10^{-4}) / 0.020 = 3.19 \rho F \]

(e) \[ R = \frac{d}{\sigma \mathbf{A}} = 0.020 / [10^{-12} (16 \times 10^{-4})] = 1.25 \times 10^{13} \Omega \]