Problem 1. (25 pts.) The magnetic field phasor of a uniform plane wave propagating at a velocity of $2.2 \times 10^8$ m/s through a nonmagnetic medium is given by

$$\vec{H} = 7e^{-j3t} \hat{x} \text{ (mA/m)}$$

Determine
(a.) the wavelength.
(b.) the frequency $f$ of the wave.
(c.) the relative permittivity of the medium.
(d.) the intrinsic impedance of the medium.
(e.) the instantaneous electric field of the wave.

Problem 2. (25 pts.) A plane wave in air (region 1, $x < 0$) is normally incident on lossless nonmagnetic medium (region 2, $x > 0$) with $\varepsilon_r = 5$. The electric field of the incident wave is given by

$$\vec{E}' = 10 \cos(\pi 10^9 t - \beta_x x) \hat{y} \text{ (V/m)}$$

Determine
(a.) the reflection and transmission coefficients.
(b.) the standing wave ratio in the air region.
(c.) the phasor reflected magnetic field.
(d.) the instantaneous transmitted electric field.
(e.) the vector average power density in the air region.

Problem 3. (25 pts.) A 300 Ω lossless transmission line (length $l = 4.25\lambda$) is terminated with a resistance of 210 Ω. The transmission line is driven by generator with

$$\vec{V}_g = 100 \angle 0 \text{ (V)} \quad Z_g = 300 \Omega$$

Determine
(a.) the input impedance seen looking into the transmission line input terminals.
(b.) the phasor voltage at the input terminals of the transmission line.
(c.) the time-average power delivered to the transmission line input terminals.
(d.) the magnitude of the voltage at the load.

Problem 4. (25 pts.) [Smith chart] A lossless 50 Ω transmission line of length $l = 1.4\lambda$ is terminated by an unknown impedance. The input impedance measured at the transmission line input is $Z_{in} = 25 + j35 \Omega$. Using the Smith chart, determine

(a.) the standing wave ratio on the transmission line.
(b.) the load impedance.
(c.) the complex reflection coefficient at the load.
(d.) the distance from the load to the first voltage maximum.
1. (a) \( \frac{q}{B} = \frac{2\pi}{3} = \frac{2\pi}{3} \) \( 2.09 \text{m} \)

(b) \( \omega = \frac{\omega}{c} \), \( \omega = \beta \mu = 3 (2.2 \times 10^8) = 6.6 \times 10^8 \), \( f = \frac{\omega}{2\pi} = 6.6 \times 10^8 \frac{\pi}{2\pi} = 1.05 \text{MHz} \)

(c) \( \beta = \frac{\omega}{c} \), \( \beta = \sqrt{1 - \frac{\omega}{c}^2} \), \( \beta = \left( \frac{\beta c}{\omega} \right)^2 = \left( \frac{6.6 \times 10^8}{3 \times 10^8} \right)^2 = 1.86 \)

(d) \( \lambda = \frac{1}{\beta c} = \frac{377}{1.86} = 200 \text{m} \)

(e) \( E^2 = 0.0076 e^{-1.33 \text{e} (-y)} \), \( E^2 = R_0 \frac{E^2}{E^2} e^{j\omega t} \), \( E^2 = \frac{-1.97 \cos (6.6 \times 10^8 t - 3\pi)}{\sqrt{3}} \) \( \frac{\text{W/m}^2}{\text{m}^2} \)

2. \( \beta_1 = \frac{\omega}{c} = \frac{\pi 10^8}{2 \times 10^8} = 10.47 \frac{\text{rad}}{m} \), \( \beta_2 = \frac{\omega}{c} \sqrt{1 - \frac{\omega}{c}^2} = 10.47 \frac{\text{rad}}{m} = \frac{6.41}{m} \)

(a) \( \tau = \frac{n_1 - n_0}{n_2 + n_1} = \frac{\frac{\sqrt{6}}{2}}{1 + \frac{\sqrt{6}}{2}} = -0.382 \), \( \tau = 1 + \tau = 0.618 \)

(b) \( \frac{x}{x_0} = \frac{1 + 101}{1 - 101} = \frac{2.24}{m} \)

(c) \( \frac{E^2}{E^2} = 10 \epsilon_0 e^{j0.47x} \), \( \frac{E^2}{E^2} = 10 \epsilon_0 e^{j0.47x} \), \( \frac{E^2}{E^2} = 10 \epsilon_0 e^{j0.47x} \)

(d) \( \frac{E^2}{E^2} = 10 \epsilon_0 e^{j25.41x} \), \( \frac{E^2}{E^2} = 10 \epsilon_0 e^{j25.41x} \), \( \frac{E^2}{E^2} = 10 \epsilon_0 e^{j25.41x} \)

(e) \( \frac{E^2}{E^2} = 10 \epsilon_0 e^{j13.3x} \), \( \frac{E^2}{E^2} = 10 \epsilon_0 e^{j13.3x} \), \( \frac{E^2}{E^2} = 10 \epsilon_0 e^{j13.3x} \)

3. (a) \( z_z = \frac{z_z - 3z_z}{z_z + 2z_z} \), \( \beta l = \frac{2\pi}{4.25} = 8.5 \pi \), \( \tan \beta l \rightarrow \infty \)

(b) \( \frac{R_0}{R_0} = \frac{300^2}{210} = \frac{429}{R_0} \), \( \frac{R_0}{R_0} = \frac{429}{R_0} \), \( \frac{R_0}{R_0} = \frac{429}{R_0} \)

(c) \( \rho_{0z} = \frac{1}{2} \frac{R_0}{R_0} \frac{\rho_{0z}}{R_0} \frac{\rho_{0z}}{R_0} \), \( \frac{\rho_{0z}}{R_0} = \frac{1}{2} \frac{R_0}{R_0} = \frac{1}{2} \frac{R_0}{R_0} = \frac{(58.25)^2}{429} = 4.04 \text{W} \)

(d) \( R_{\text{c}} = 20 \), \( |V|_{\text{min}} \) at load \( (z = l) \), \( |V|_{\text{min}} \) at input \( (z = 0) \), \( \frac{10^{10}}{10^{10}} = \frac{1 + \frac{2}{1}}{1 + \frac{2}{1}} \)

\( \tau = \frac{2 \times 10}{210 + 200} = 0.176 \), \( \frac{|V|_{\text{load}}}{|V|_{\text{min}}} = \frac{|V|_{\text{load}}}{|V|_{\text{min}}} = 58.25/1.429 = 41.24 \text{W} \)
(a) \[ z_{in} = \frac{Z_{in}}{Z_o} = \frac{25 + j35}{50} = 0.5 + j0.7 \quad (100.5^\circ) \quad s = 3.2 \]

(b) \[ 0.4\lambda \times \frac{720^\circ}{1/\lambda} = 288^\circ \quad \text{From } z_{in}, \text{ rotate toward load (CCW) } 288^\circ \text{ to find } z_L \]
\[ 100.5^\circ + 288^\circ = 388.5^\circ \quad (28.5^\circ) \]
\[ z_L = 2.05 + j1.42 \quad \Rightarrow \quad Z_L = Z_o z_L = 50(2.05 + j1.42) = (102 + j71) \Omega \]

(c) \[ \Gamma_L = 0.52 \angle 28.5^\circ \]

(d) \[ V_{\text{max}} \quad \text{(rightmost point on VSWR circle)} \]
\[ \text{Rotate from } z_L \text{ toward generator (CW) to } V_{\text{max}} \quad (28.5^\circ) \]
\[ 28.5^\circ \times \frac{1/\lambda}{720^\circ} = 0.040 \lambda \]