SYNCHRONOUS MACHINES

The geometry of a synchronous machine is quite similar to that of the induction machine. The stator core and windings of a three-phase synchronous machine are practically identical to that of a three-phase induction machine. The function of the synchronous machine stator is to provide a rotating mmf to the rotor, just as the stator of the induction machine. The synchronous machine rotor, on the other hand, is different than that of the induction machine.

The rotor of the synchronous machine is a rotating electromagnet with the same number of poles as the stator. The poles of the synchronous machine rotor are created by the rotor windings which carry DC currents. Thus, the synchronous machine requires simultaneous AC and DC excitation of the stator (*armature*) windings and the rotor (*field*) windings, respectively. The magnetic moments associated with the poles of the rotor follow the magnetic moments of the stator-generated mmf which rotates at the synchronous speed. In other words, the magnetic fields of the stator and the rotor tend to align themselves. Therefore, under steady state conditions given a constant frequency AC source, the machine speed (n) of a synchronous machine is equal to the synchronous speed (n_s) defined by

$$n = n_s = 120 \frac{f}{p} \quad (\text{rpm})$$

where f is the frequency of the AC signal at the stator, and p is the number of poles in the synchronous machine. Thus, the fundamental difference between a synchronous machine and an induction machine is that the rotor currents of the induction machine are induced while those of the synchronous machine are not.

There are fundamentally two types of rotors used in synchronous machines: *salient pole* rotors and *cylindrical* (or *non-salient* pole) rotors (see Figure 3-12, p. 170). These rotors are each well-suited for different applications based on their physical characteristics.

Synchronous Machine Rotor Types

1. *Salient pole rotor* - the individual rotor poles protrude from the center of the rotor, characterized by concentrated windings, non-uniform air gap, larger rotor diameters, used in applications requiring low machine speed and a large number of machine poles (example - hydroelectric generation).

2. *Cylindrical rotor* - the individual rotor poles are produced using a slotted cylindrical rotor, characterized by distributed windings, nearly-uniform air gap, smaller rotor diameters, used in applications requiring high machine speed and a small number of machine poles, typically 2 or 4 poles (example - steam or gas turbine generators).

The cylindrical rotor is typically a solid piece of steel (made from a single forging) for reasons of strength given the high rotational speeds to which the rotor is subjected. The salient pole rotor does not provide the mechanical strength necessary for these high-speed applications. Also, the salient pole rotor presents too much wind resistance when rotating at high speeds.

The DC current required for the rotor is typically provided by an external DC source (commonly referred to as an *exciter*) that is connected to the rotor windings by means of conducting rings (slip rings) that are mounted concentrically on the machine shaft (the slip rings are electrically insulated from the shaft). The stationary contact required to connect the DC source with these slip rings is achieved by means of carbon brushes that make physical contact with the slip rings as they rotate. The carbon brushes make good electrical contact with low friction.

The DC rotor current can also be provided by a rectifying source (converts AC to DC) mounted directly to the machine shaft. This type of configuration is known as *brushless excitation*.

SYNCHRONOUS GENERATOR
(SYNCHRONOUS MACHINE EQUIVALENT CIRCUIT)

The synchronous machine can be operated as a motor or a generator, but the most common application of the synchronous machine is in the power industry as a three-phase generator. The synchronous generator is also sometimes referred to as an *alternator*.

In a synchronous generator, the magnetic field produced by the DC current in the rotor is static in nature (magnetostatic field). However, if the rotor is set in motion by some external force (wind, water, turbine, etc.), the rotating magnetic field produced by the synchronous generator rotor looks like the rotational mmf produced by the AC current in the stator of the induction motor. This rotating mmf changes the magnetic flux with time through the stator windings inducing an emf in the stator terminals according to Faraday's law. The frequency f of the voltage produced at the stator windings is directly related to the mechanical speed of the rotor rotation (n_m) in rpm by

$$f = \frac{n_m p}{120} \quad \text{(Hz)}$$

where p is the number of poles. As previously shown for the induction motor, the induced emf in the stator terminals can be related to the total magnetic flux ψ_m in the magnetic circuit formed by the rotor/air gap/stator according to

$$\boldsymbol{V}_1 = 4.44 N_1 f \psi_m K_{W1}$$

where N_1 is the number of turns in the stator winding and K_{W1} is the stator winding factor. The total magnetic flux through the stator winding is a superposition of the fluxes due to the stator and rotor currents.

$$\psi_m = \psi_{ma} + \psi_{mf}$$

ψ_{ma} - magnetic flux due to stator (armature) current

ψ_{mf} - magnetic flux due to rotor (field) current

Inserting the total stator flux equation into the stator emf equation yields two voltage components which may be defined as

$$\boldsymbol{V}_1 = \boldsymbol{V}_a + \boldsymbol{V}_f$$

where

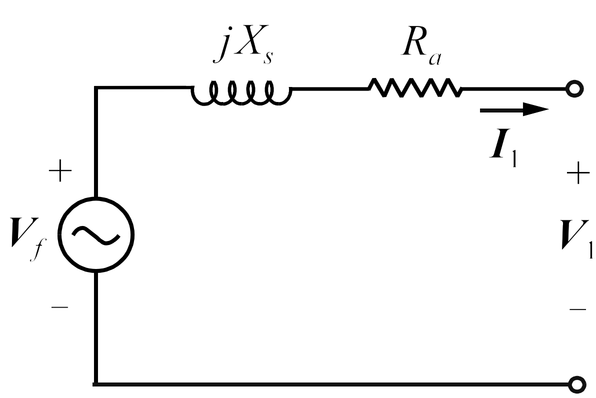
$\boldsymbol{V}_a$ - phasor stator voltage component due to ψ_{ma}

$\boldsymbol{V}_f$ - phasor stator voltage component due to ψ_{mf}

When the synchronous machine is operated as a generator, the voltage component $\boldsymbol{V}_f$ represents the generated voltage while the voltage $\boldsymbol{V}_a$ represents the response of the stator to the generation process. Solving for the generator voltage gives

$$\boldsymbol{V}_f = \boldsymbol{V}_1 - \boldsymbol{V}_a$$

In the equation above, the $-\boldsymbol{V}_a$ term represents the voltage drop in the generated voltage due to leakage magnetic flux, magnetization of the synchronous machine core, and losses in the stator windings. This relationship can be represented by a simple equivalent circuit as shown below.



Synchronous machine equivalent circuit

$$\boldsymbol{V}_f = \boldsymbol{V}_1 + \boldsymbol{I}_1 \boldsymbol{Z}_s$$

The impedance $\boldsymbol{Z}_s = R_a + jX_s$ is defined as the *synchronous impedance* where R_a is the *armature effective resistance* and X_s is defined as the *synchronous reactance*. The synchronous reactance contains two components: the leakage reactance X_l and the magnetization reactance X_m.

It is unusual for a synchronous generator to be used to supply a single load. In power systems applications, large numbers of individual synchronous generators are connected to the *power grid* which is sometimes called an *infinite bus*. Large synchronous generators are connected to the power transmission grid located at various locations. These generators are attached to the grid via transformers since the generator output voltage must be stepped up to a higher level for efficient transmission. Various load centers are also connected to the grid throughout the system. These load centers are also attached to the grid via transformers since the load voltage must be stepped down from the transmission voltage level.

Given the large number of generators connected to the power grid, the voltage level and frequency on the power grid stays very stable, even when generators and loads are brought online and taken offline. This is the primary advantage of the infinite bus.

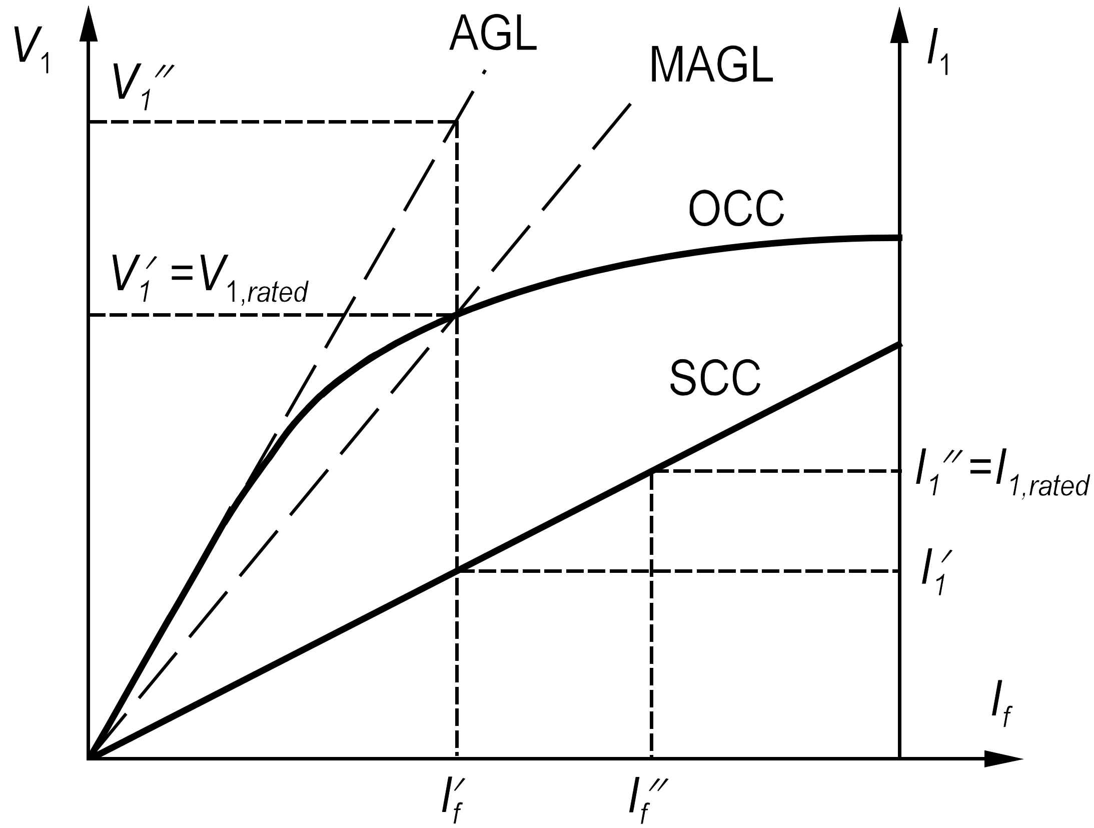
DETERMINATION OF THE SYNCHRONOUS MACHINE EQUIVALENT CIRCUIT PARAMETERS
(OPEN-CIRCUIT AND SHORT-CIRCUIT TESTS)

The components of the synchronous reactance can be determined by performing two tests on the synchronous machine: the open-circuit test and the short-circuit test.

<u>Open-Circuit Test</u> With the stator windings open-circuited, the synchronous machine is driven at synchronous speed while the field current I_f is varied. The open-circuit voltage V_1 across the stator windings is measured. This test provides data for a plot of V_1 vs. I_f. This plot is known as the *open-circuit characteristic* (OCC) and represents the variation of the generator voltage with respect to the field current.

<u>Short-Circuit Test</u> With the stator input terminals short-circuited (the stator windings connected in parallel), the synchronous machine is driven at synchronous speed while the field current I_f is varied. The current I_1 in each of the three stator windings is measured and an average value is determined. This test provides data for a plot of I_1 vs. I_f. This plot is known as the *short-circuit characteristic* (SCC) and represents the variation of the armature (stator) current with respect to the field current.

The OCC will be nonlinear due to the saturation of the magnetic core at higher levels of field current. The SCC will be linear since the magnetic core does not saturate under short-circuit conditions.



The armature effective resistance R_a is often very small and may be neglected in most problems. If this value is included in the analysis, it will typically be provided.

<u>Determination of the Synchronous Reactance</u>

According to the connections made in the open-circuit and short-circuit tests, the value of the synchronous impedance is the ratio of the open-circuit test stator voltage to the short-circuit test stator current. This ratio should be evaluated at a common value of field current such as I_f' shown in the plot above. Note that this value of field current is the value at which the OCC passes through the rated voltage of the synchronous machine.

The synchronous reactance will have a different value depending on whether or not the magnetic core is saturated. Note that the core is saturated on the OCC at the field current value of I_{f1}. If the magnetic core were unsaturated at this field current value, the OCC would continue along the linear portion of the curve which has been extended on the plot as the *air gap line* (AGL). The unsaturated value of the synchronous impedance is found according to

$$Z_{s,unsat} = \frac{V_1''}{I_1'} = \sqrt{R_a^2 + X_{s,unsat}^2}$$

such that the synchronous reactance is

$$X_{s,unsat} = \sqrt{\left(\frac{V_1''}{I_1'}\right)^2 - R_a^2}$$

If R_a is negligible, the unsaturated synchronous reactance is

$$X_{s,unsat} = \frac{V_1''}{I_1'}$$

Assuming the generator is connected to an infinite bus, the synchronous reactance at saturation is determined using the value of V_1 that occurs on the *modified air gap line* (MAGL). When the synchronous generator is connected to the infinite bus, its terminal voltage is raised to the rated value (where the core is saturated). After connection to the infinite bus, the terminal voltage of the generator will remain constant. If the field current is now changed, the generated voltage will change, not along the OCC curve, but along the modified air gap line. Thus, the synchronous impedance at saturation is given by

$$Z_{s,sat} = \frac{V_1'}{I_1'} = \frac{V_{1rated}}{I_1'} = \sqrt{R_a^2 + X_{s,sat}^2}$$

The saturated synchronous reactance is then

$$X_{s,sat} = \sqrt{\left(\frac{V_{1rated}}{I'_1}\right)^2 - R_a^2}$$

If R_a is negligible, the unsaturated synchronous reactance is

$$X_{s,sat} = \frac{V_{1rated}}{I'_1}$$

Example (Synchronous machine equivalent model)

The following data is obtained for a three-phase 10 MVA, 14 kV wye-connected synchronous machine (all voltages are line-to-line). The armature resistance is 0.07 Ω per phase.

I_f (A)	OCC (kV)	SCC (A)	AGL (kV)
100	9.0		
150	12.0		
200	14.0	490	18
250	15.3		
300	15.9		
350	16.4		

(a.) Find the unsaturated and saturated values of the synchronous reactance in Ω and pu.

(b.) Find the field current required if the synchronous generator is connected to an infinite bus and delivers rated MVA at 0.8 lagging power factor.

(c.) If the generator, operating as in part (b.), is disconnected from the infinite bus without changing the field current, find the terminal voltage.

The base quantities for pu calculations are:

$$S_{base} = 10^7 \text{ VA}$$
$$V_{base} = V_{1\text{rated}} = 14000/\sqrt{3} = 8083 \text{ V}$$
$$I_{base} = S_{base}/3V_{base} = 10^7 \text{ VA}/[3(8083)] \text{ V} = 412.4 \text{ A}$$
$$Z_{base} = V_{base}/I_{base} = 8083/412.4 = 19.60\ \Omega$$

(a.)

$$V_1'' = \frac{18000}{\sqrt{3}} = 10392 \text{ V} \qquad I_1' = 490 \text{ A}$$

$$X_{s,unsat} = \sqrt{\left(\frac{V_1''}{I_1'}\right)^2 - R_a^2} = \sqrt{\left(\frac{10392}{490}\right)^2 - 0.07^2} = 21.2\ \Omega$$

$$X_{s,unsat,pu} = \frac{21.2}{19.6} = 1.08 \text{ pu}$$

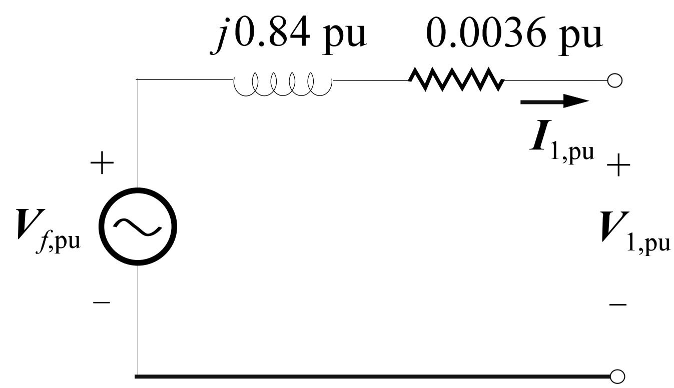
$$V_1' = V_{1rated} = 8083 \text{ V} \qquad I_1' = 490 \text{ A}$$

$$X_{s,sat} = \sqrt{\left(\frac{V_1'}{I_1'}\right)^2 - R_a^2} = \sqrt{\left(\frac{8083}{490}\right)^2 - 0.07^2} = 16.5\ \Omega$$

$$X_{s,sat,pu} = \frac{16.5}{19.6} = 0.84 \text{ pu}$$

$$R_{a,pu} = \frac{0.07}{19.6} = 0.0036 \text{ pu}$$

(b.) Given that the machine is operating at rated load, the machine can be assumed to be operating in saturation. Thus, the equivalent circuit containing the saturated synchronous reactance is used.



$$\boldsymbol{V}_{1,\text{pu}} = 1\angle 0^o \text{ pu}$$

$$PF = 0.8 \qquad \theta_v - \theta_i = \cos^{-1}(0.8) = 36.87^o$$

$$I_{1,\text{pu}} = 1\angle -36.87^o \text{ pu}$$

$$\boldsymbol{V}_{f,\text{pu}} = \boldsymbol{V}_{1,\text{pu}} + \boldsymbol{I}_{1,\text{pu}}\boldsymbol{Z}_{s,\text{pu}}$$

$$= 1\angle 0^o + (1\angle -36.87^o)(0.84 + j0.0036)$$

$$= 1.649\angle 23.97^o \text{ pu}$$

$$\boldsymbol{V}_f = V_{base}\,\boldsymbol{V}_{f,\text{pu}} = (8083 \text{ V})\,(1.649\angle 23.97^o \text{ pu})$$

$$= 13.32\angle 23.97^o \text{ kV}$$

The corresponding field current is found from the MAGL (the MAGL is an open-circuit characteristic where $\boldsymbol{V}_f = \boldsymbol{V}_1$). The equation for the MAGL (slope = 14000V/200A = 70 V/A) is

$$V_1 = 70 I_f$$

such that the field current for $\boldsymbol{V}_f = \boldsymbol{V}_1 = 23.09$ kV is

$$I_f = \frac{V_1}{70} = \frac{\sqrt{3}\,(13320)}{70} = 329.9 \text{ A}$$

(c.) If the generator is disconnected from the infinite bus, then we jump from the MAGL to the OCC. For the same field current (I_f = 329.9 A), the corresponding terminal voltage is

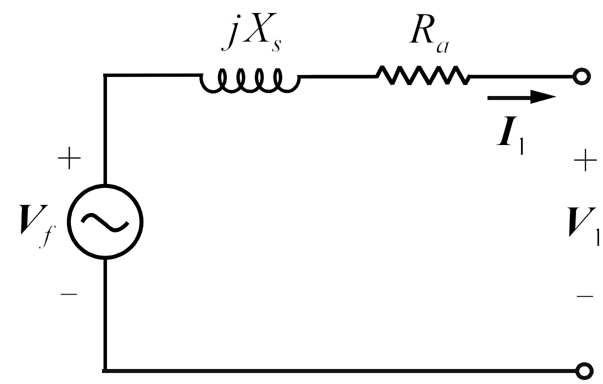
$$V_1 = 16.25\text{ kV}$$

SYNCHRONOUS MACHINE POWER AND TORQUE

The complex power at the terminals of the synchronous machine can be determined using the equivalent circuit. For convenience, the machine terminals are used as the phase reference such that

$$\boldsymbol{V}_1 = V_1 \angle 0^o \qquad \boldsymbol{V}_f = V_f \angle \delta \qquad \boldsymbol{Z}_s = Z_s \angle \theta_s$$

where δ is the phase angle of the generating voltage and θ_s is the phase angle of the synchronous impedance. The complex power at the terminals of the synchronous machine is



$$\boldsymbol{S} = \boldsymbol{V}_1 \boldsymbol{I}_1^*$$

From the equivalent circuit, the current $\boldsymbol{I}_1$ is

$$\boldsymbol{I}_1 = \frac{\boldsymbol{V}_f - \boldsymbol{V}_1}{\boldsymbol{Z}_s} = \frac{V_f}{Z_s}\angle(\delta - \theta_s) - \frac{V_1}{Z_s}\angle(-\theta_s)$$

The conjugate of $\boldsymbol{I}_1$ is

$$\boldsymbol{I}_1^* = \frac{V_f}{Z_s}\angle(\theta_s - \delta) - \frac{V_1}{Z_s}\angle\theta_s$$

The resulting per phase complex power is

$$\boldsymbol{S} = \boldsymbol{V}_1 \boldsymbol{I}_1^* = \frac{V_1 V_f}{Z_s} \angle (\theta_s - \delta) - \frac{V_1^2}{Z_s} \angle \theta_s$$

The real power and reactive power are

$$P = \frac{V_1 V_f}{Z_s} \cos(\theta_s - \delta) - \frac{V_1^2}{Z_s} \cos\theta_s$$

$$Q = \frac{V_1 V_f}{Z_s} \sin(\theta_s - \delta) - \frac{V_1^2}{Z_s} \sin\theta_s$$

If the effective armature resistance is neglected ($R_a = 0$), the magnitude of the synchronous impedance becomes the magnitude of the synchronous reactance ($Z_s = X_s$) with $\theta_s = 90^\circ$ and the per-phase expressions for P and Q reduce to

$$P = \frac{V_1 V_f}{X_s} \sin\delta$$

$$Q = \frac{V_1 V_f}{X_s} \cos\delta - \frac{V_1^2}{X_s}$$

For a three-phase synchronous machine, we have

$$P_{3\phi} = \frac{3 V_1 V_f}{X_s} \sin\delta = P_{\max} \sin\delta$$

$$Q_{3\phi} = P_{\max} \cos\delta - \frac{3 V_1^2}{X_s}$$

where

$$P_{\max} = \frac{3 V_1 V_f}{X_s}$$

The stator losses have been neglected by assuming $R_a = 0$, so that the real power at the synchronous machine terminals is equal to the air gap power which is equal to the developed torque times the angular velocity of the machine.

$$P_{3\phi} = T\omega_s$$

$$T = \frac{P_{3\phi}}{\omega_s} = \frac{3\,V_1 V_f}{\omega_s X_s}\sin\delta = \frac{P_{\max}}{\omega_s}\sin\delta = T_{\max}\sin\delta$$

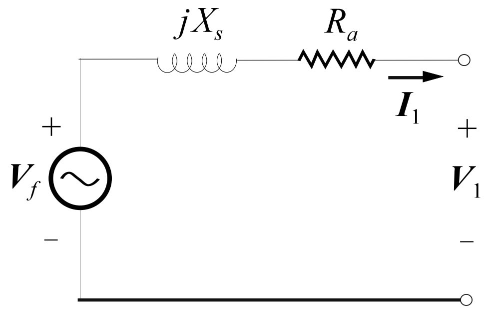
where

$$T_{\max} = \frac{3\,V_1 V_f}{\omega_s X_s}$$

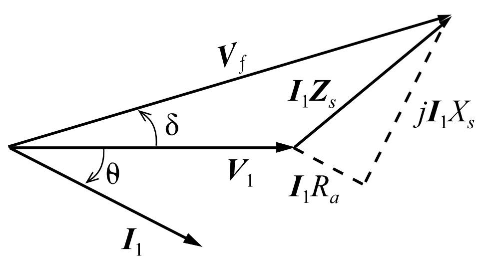
Note that the power and the torque vary as the sin of the angle δ which is known as the *power angle* or *torque angle*. The synchronous machine will stay at synchronous speed if it is loaded gradually up to the limit of P_{max} for a generator or T_{max} for a motor. But the machine loses synchronism if the power angle goes greater than 90^o. The value of maximum torque T_{max} is known at the *pull-out torque*. Note that the value of the pull-out torque can be increased by increasing the value of V_f. Thus, a synchronous motor that tends to lose synchronism due to excessive torque can be brought back into synchronism by increasing the field current. A synchronous generator may lose synchronism because the prime mover tends to rotate the machine at speeds above the synchronous speed. The synchronous generator can be brought back into synchronism by increasing the field current, which increases the counter-torque, and slows the machine down to synchronous speed.

PHASOR DIAGRAMS FOR SYNCHRONOUS MACHINES

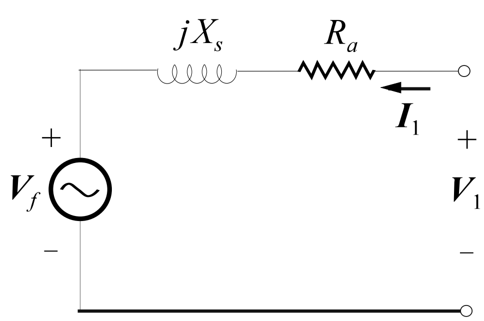
Synchronous Generator



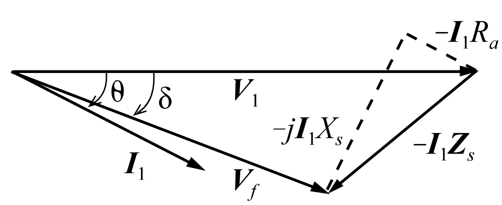
$$\boldsymbol{V_f} = \boldsymbol{V_1} + \boldsymbol{I_1 Z_s}$$



Synchronous Motor



$$\boldsymbol{V_f} = \boldsymbol{V_1} - \boldsymbol{I_1 Z_s}$$



Example (Synchronous generator / complex power)

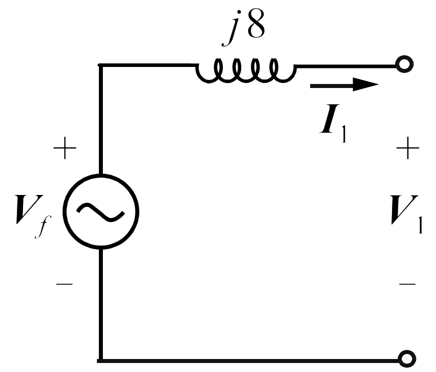
A three-phase 5kVA, 208 V, four-pole, 60 Hz wye-connected synchronous machine has negligible stator winding resistance and a synchronous reactance of 8 Ω per phase at rated voltage. The machine is operated as a generator in parallel with a three-phase 208V, 60 Hz power supply.

(a.) Determine the generator voltage and the power angle when the machine is delivering rated kVA at 0.8 PF lagging. Draw the phasor diagram.

(b.) With the same field current as in part (a.), the power of the prime mover is slowly increased. Determine the values of the stator current, power factor, real power and reactive power at the maximum power transfer condition.

$$\boldsymbol{V}_{1rated} = \frac{V_{LL}}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120\text{ V}$$

$$\boldsymbol{I}_{1rated} = \frac{S}{\sqrt{3}\,V_{LL}} = \frac{5000}{\sqrt{3}\,(208)} = 13.88\text{ A}$$



$$\boldsymbol{V}_1 = 120\angle 0^o\text{ V}$$

$$PF = 0.8\ \text{lagging}$$

$$\theta_v - \theta_i = \cos^{-1}(0.8) = 36.87^o$$

$$I_1 = 13.88\angle -36.87^o\text{ A}$$

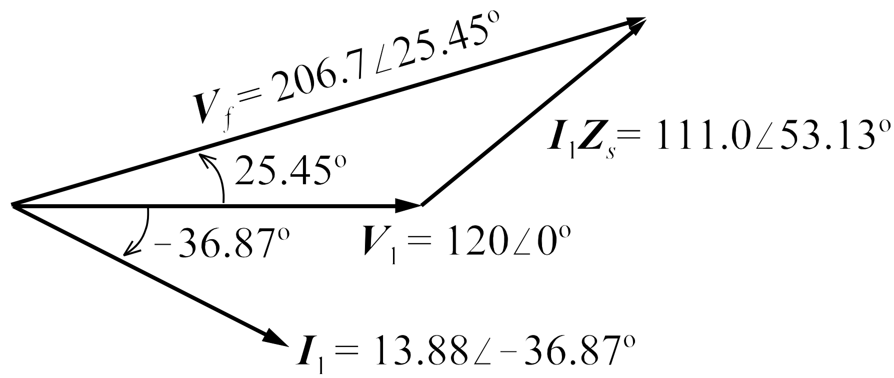
$$\boldsymbol{V}_f = \boldsymbol{V}_1 + \boldsymbol{I}_1\boldsymbol{Z}_s$$

$$= 120\angle 0^o + (13.88\angle -36.87^o)(8\angle 90^o)$$

$$= 120\angle 0^o + 111.0\angle 53.13^o$$

$$= 206.7\angle 25.45^o\text{ V} = V_f\angle\delta$$

$$V_f = 206.7\ \text{V} \qquad \delta = 25.45^o$$



(b.) V_1, V_f stay constant, δ changes,
Maximum power condition ⇒ δ = 90°

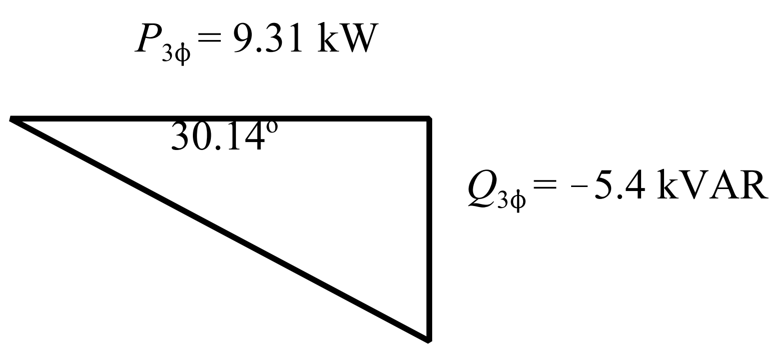
$$V_f = 206.7\angle 90^o$$

$$P_{3\phi} = P_{max}\sin\delta = P_{max} = \frac{3V_1V_f}{X_s} = \frac{3(120)(206.7)}{8} = 9.31\ \text{kW}$$

$$Q_{3\phi} = P_{max}\cos\delta - \frac{3V_1^2}{X_s} = -\frac{3V_1^2}{X_s} = -\frac{3(120)^2}{8} = -5.4\ \text{kVAR}$$

$$\boldsymbol{I}_1 = \frac{\boldsymbol{V}_f - \boldsymbol{V}_1}{jX_s} = \frac{206.7\angle 90^o - 120\angle 0^o}{8\angle 90^o} = 29.88\angle 30.14^o\ \text{A}$$

$$PF = \cos(30.14^o) = 0.865\ \text{leading}$$

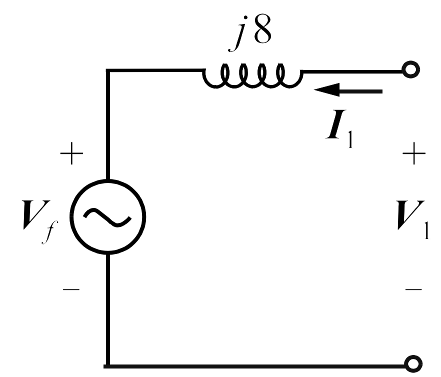


Example (Synchronous motor)

The synchronous machine defined in the previous example is operated as a synchronous motor when fed from a three-phase 208 V, 60 Hz supply. The field excitation is adjusted so that power factor is unity when the machine draws 3 kW from the supply.

(a.) Determine the excitation voltage and the power angle. Draw the phasor diagram.

(b.) If the field excitation is held constant and the shaft load is slowly increased, determine the maximum torque (*pull-out torque*) that the motor can deliver.



(a.) $\boldsymbol{V}_1 = 120\angle 0^o$ V

$PF = 1.0 \qquad \theta_v - \theta_i = 0^o$

$P = 3V_1I_1 = 3(120)I_1 = 3$ kW

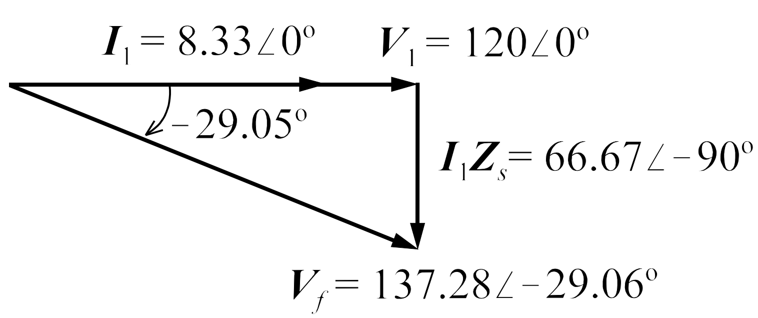
$$I_1 = \frac{3000}{3(120)} = 8.33 \text{ A}$$

$$\boldsymbol{V}_f = \boldsymbol{V}_1 - \boldsymbol{I}_1\boldsymbol{Z}_s = 120\angle 0^o - (8.33\angle 0^o)(8\angle 90^o)$$

$$= 120\angle 0^o - 66.67\angle 90^o$$

$$= 137.28\angle -29.06^o \text{ V} = V_f\angle\delta$$

$V_f = 137.28$ V $\qquad \delta = -29.06^o$



(b.) V_1, V_f stay constant, δ changes,
Maximum power condition ⇒ $\delta = 90^{\circ}$

$$V_f = 137.28 \angle 90^{o}$$

$$P_{\max} = \frac{3\, V_1 V_f}{X_s} = \frac{3(120)(137.28)}{8} = 6.18\ \text{kW}$$

$$T_{\max} = \frac{P_{\max}}{\omega_s} \qquad \omega_s = \frac{n_s}{60} 2\pi \qquad n_s = 120\frac{f}{p}$$

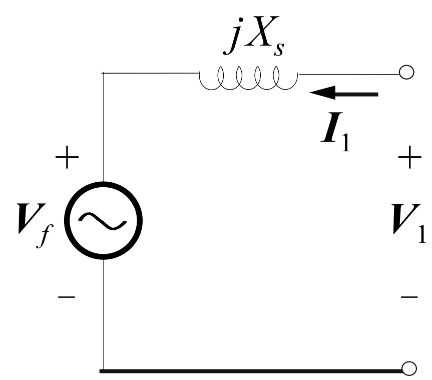
$$\omega_s = 4\pi\frac{f}{p} = 4\pi\frac{60}{4} = 188.5\ \text{rad/s}$$

$$T_{\max} = \frac{6180}{188.5} = 32.8\ \text{N-m}$$

POWER FACTOR CONTROL WITH SYNCHRONOUS MOTORS

The power factor of a synchronous machine is easily controlled by adjusting the machine field current. Leading or lagging power factors are achievable. Thus, synchronous motors with a leading power factor can be utilized in combination with induction motors (lagging power factors) to improve the overall power factor of the load combination.

The power factor characteristic of a synchronous motor as a function of field current can be demonstrated by examining the phasor diagram of the motor. Consider a three-phase synchronous motor connected to an infinite bus and delivering a required output power $P_{3\varphi}$ as shown in the per-phase equivalent circuit for the synchronous motor shown below, where the stator resistance has been neglected. The real three-phase output power of the motor can be defined according to the simple terminal characteristics of the motor at the stator input in the per-phase equivalent circuit.



$$P_{3\varphi} = 3\, V_1 I_1 \cos(\theta_v - \theta_i)$$

Since the stator voltage V_1 is constant (given the infinite bus connection), the product terminal current and power factor must remain constant according to

$$I_1 \cos(\theta_v - \theta_i) = I_1 PF = \text{constant}$$

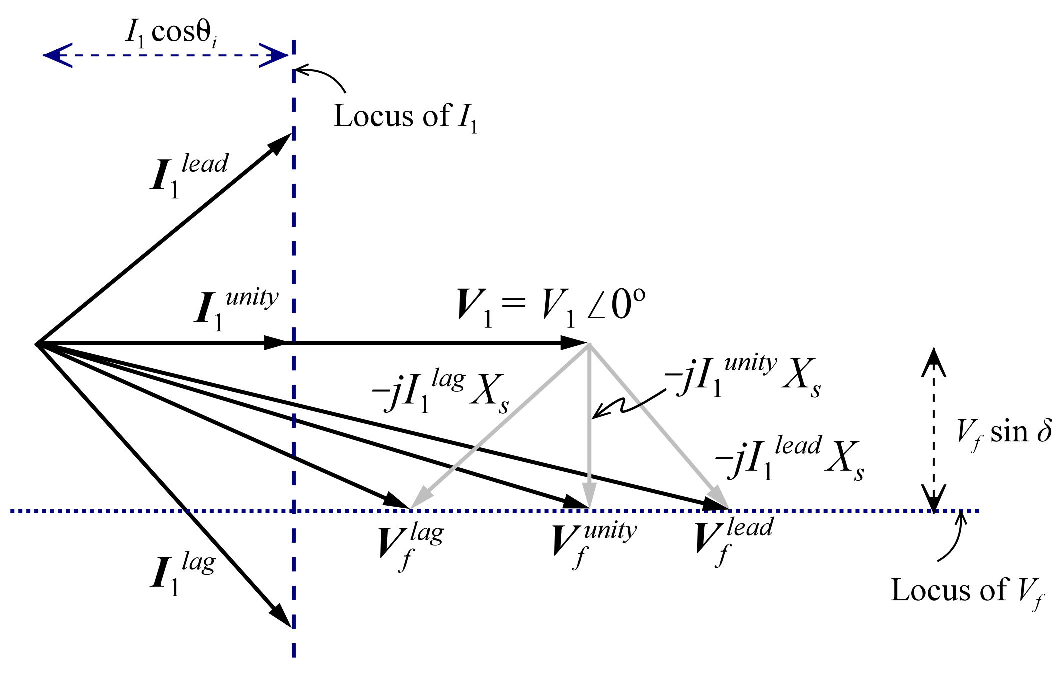
to deliver the required value of real power. If we use the stator voltage as the reference phase ($\theta_v = 0$), then the stator current must satisfy

$$I_1 \cos\theta_i = \text{constant}$$

for the required output power. On a phasor diagram, this requirement on the stator current corresponds to a vertical locus as shown below (the real part of the $\boldsymbol{I}_1$ phasor is constant). The circuit relationship between the stator current and voltage, as seen in the per-phase equivalent circuit is

$$\boldsymbol{V}_f = \boldsymbol{V}_1 - j\boldsymbol{I}_1 X_s$$

If we consider arbitrary stator currents for leading, lagging and unity power factor conditions, we can draw the stator current phasors, and determine the direction of the resulting $-j\boldsymbol{I}_1 X_s$ phasor (which should point in the direction of the corresponding $\boldsymbol{I}_1$ vector rotated clockwise by 90°). Adding the $-j\boldsymbol{I}_1 X_s$ phasor to the $\boldsymbol{V}_1$ phasor yields the $\boldsymbol{V}_f$ phasor.



The magnitude of the $\boldsymbol{V}_f$ phasor can be determined from the three-phase synchronous motor power relationship in terms of the power angle:

$$P_{3\varphi} = \frac{3\,V_1 V_f}{X_s}\sin\delta = P_{max}\sin\delta$$

Again, noting that V_1 and X_s are constant, then

$$V_f \sin\delta = \text{constant}$$

Since the angle δ represents the phase angle of the $\boldsymbol{V_f}$ phasor, the constraint above means that the vertical component (imaginary part) of the $\boldsymbol{V_f}$ phasor is constant, which defines a horizontal locus for V_f.

The excitation voltage in a synchronous motor V_f changes linearly with the field current I_f. Thus, the three $\boldsymbol{V_f}$ phasors in the previous phasor diagram represent three different values of field current, demonstrating that adjustments to the field current can result in different power factor conditions. According to the magnitude of the $\boldsymbol{V_f}$ phasors illustrated in the phasor diagram, we find

$$I_f^{lag} < I_f^{unity} < I_f^{lead}$$

According to the phasor diagram, the stator current magnitude is minimum at unity power factor, and grows larger for leading or lagging power factors. The power factor characteristics of a synchronous motor are typically defined using the terms *normal excitation*, *underexcitation*, and *overexcitation*.

Synchronous motor mode	**PF**	**Field current**	**Stator current**
Underexcitation	lagging	low	large
Normal excitation	unity	midrange	minimum
Overexcitation	leading	high	large

Note that the power factor and stator current have inverse relationships. The power factor is maximum when the stator current is minimum.

Example (Synchronous motor, power factor correction)

The load at a factory consists of several three-phase induction motors and a single synchronous motor. The induction motors absorb 500 kVA at 0.8 PF lagging while the synchronous motor (4 kV, 400 kVA) absorbs 300 kVA at unity power factor.

(a.) Determine the overall factory power factor and the input current drawn by the synchronous motor.

(b.) The synchronous motor is now overexcited to further improve the factory *PF*, assuming no change in the motor load. Determine the optimum factory *PF* given the constraint that the kVA rating of the synchronous motor should not be exceeded.

(c.) Determine the input current and *PF* for the synchronous motor in part (b.).

(a.) Induction motors:

$$\cos(\theta_v - \theta_i) = 0.8 \qquad \Rightarrow \qquad (\theta_v - \theta_i) = 36.87^\circ$$

$$P_{IM} = 500 \times \cos(36.87^\circ) = 400 \text{ kW}$$

$$Q_{IM} = 500 \times \sin(36.87^\circ) = 300 \text{ kVAR}$$

Synchronous motor:

$$P_{SM} = 300 \text{ kW}$$

$$Q_{IM} = 0 \text{ kVAR}$$

Factory load:

$$P_F = P_{IM} + P_{SM} = 400 + 300 = 700 \text{ kW}$$

$$Q_F = Q_{IM} + Q_{SM} = 300 + 0 = 300 \text{ kVAR}$$

$S_F = [P_F^2 + Q_F^2]^{1/2} = [700^2 + 300^2]^{1/2} = 761.6$ kVA

$\text{PF}_F = P_F / S_F = 700 / 761.6 = 0.919$ lagging

Input current for the synchronous motor:

$$S_{SM} = \sqrt{3}\, V_{LL} I_L$$

$$I_L = \frac{300000}{\sqrt{3}\, 4000} = 43.30 \text{ A}$$

(b.) Synchronous motor rating = 400 kVA

The maximum leading kVAR the synchronous motor can draw without exceeding its rating can be calculated from the power triangle.

$S_{SM} = 400$ kVA

$P_{SM} = 300$ kW

$Q_{SM} = -[S_{SM}^2 - P_{SM}^2]^{1/2} = -[400^2 - 300^2]^{1/2} = -264.6$ kVAR

Factory load:

$P_F = P_{IM} + P_{SM} = 400 + 300 = 700$ kW

$Q_F = Q_{IM} + Q_{SM} = 300 - 264.6 = 35.4$ kVAR

$S_F = [P_F^2 + Q_F^2]^{1/2} = [700^2 + 35.4^2]^{1/2} = 700.9$ kVA

$PF_F = P_F / S_F = 700 / 700.9 = 0.9987$ lagging (c.) Synchronous motor PF and current.

$$PF_{SM} = P_{SM} / S_{SM} = 300 / 400 = 0.75 \text{ leading}$$

$$S_{SM} = \sqrt{3}\, V_{LL}\, I_L$$

$$I_L = \frac{400000}{\sqrt{3}\,4000} = 57.74 \text{ A}$$

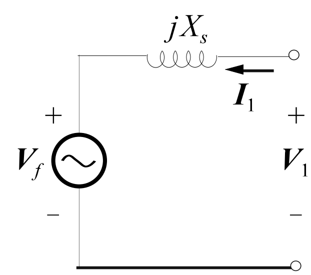
<u>Example</u> (Synchronous motor, field current variation)

A 1 MVA, 2.3 kV, 60 Hz, wye-connected three-phase synchronous machine (X_s = 5.03 Ω, stator resistance is negligible) is connected to an infinite bus and operated as a synchronous motor. The machine efficiency at rated speed is 95%.

(a.) Determine the excitation voltage and power angle when the motor operates at PF = 0.85 lagging with an output power of 500 hp.

(b.) The field current is now reduced by 40%, while the output power remains constant. Determine the stator current and power factor for the machine.

(a.)



$$P_{in} = \frac{P_{out}}{\eta} = \frac{(500)(746)}{0.95} = 392.6\text{ kW}$$

$$S_{in} = \sqrt{3}\,V_{LL} I_L = \frac{P_{in}}{PF} \quad \Rightarrow \quad I_L = \frac{392600/0.85}{\sqrt{3}(2300)} = 115.94\text{ A} = I_1$$

$$\boldsymbol{V}_1 = \frac{2300}{\sqrt{3}}\angle 0^o = 1327.9\angle 0^o\text{ V}$$

$$\boldsymbol{I}_1 = 115.94\angle -\cos^{-1}(0.85) = 115.94\angle -31.79^o\text{ A}$$

$$\boldsymbol{V}_f = \boldsymbol{V}_1 - j\boldsymbol{I}_1 X_s = 1327.9\angle 0^o - (115.94\angle -31.79^o)(5.03\angle 90^o)$$

$$= 1134.7\angle -25.90^o$$

$$V_f = 1134.7\text{ V} \qquad \delta = -25.90^o$$

(b.) $I_{fb} = 0.6 I_{fa} \quad \Rightarrow \quad V_{fb} = 0.6\, V_{fa}$

$P_a = P_b \quad \Rightarrow \quad V_{fa} \sin\delta_a = V_{fb} \sin\delta_b$

$$\delta_b = \sin^{-1}\left(\frac{\sin(-25.90^o)}{0.6} \right) = -46.72^o$$

$$\boldsymbol{I}_{1b} = \frac{\boldsymbol{V}_{fb} - \boldsymbol{V}_{1b}}{-jX_s} = \frac{(0.6)(1134.7\ \angle -46.72^o) - 1327.9 \angle\, 0^o}{5.03\ \angle -90^o}$$

$$= 197.54 \angle -60.08^o \text{ A}$$

$I_{1b} = 197.54$ A

$PF = \cos(60.08^o) = 0.499$

STARTING SYNCHRONOUS MOTORS

Unlike the induction motor, the synchronous motor is not self-starting (it cannot simply be connected to the AC supply to start). If a synchronous motor is connected directly to the AC supply, it will simply vibrate. The inertia of the rotor prevents it from locking onto the rotating stator field. Two techniques are commonly used to start an synchronous motor.

Variable-Frequency Supply - The synchronous motor can be started with a frequency converter (variable frequency output) by slowly increasing the frequency of the stator field upon startup. This allows the rotor time to overcome the inertia required for it to follow the stator field as it increases in speed. The primary drawback to this technique is the cost of the frequency converter.

Starting the Synchronous Motor as an Induction Motor - Additional windings known as *damper* windings can be added to the synchronous motor to allow it to start as an induction motor. The damper windings closely resemble the cage of an induction motor. To start the induction motor, no DC current is passed through the field winding initially. When the motor is connected to the supply, the synchronous motor will start like an induction motor as currents are induced in the damper windings. The motor will increase speed until it reaches a speed slightly less than synchronous speed. At that time, the DC field current is applied to the rotor. Since the rotor is closely following the stator field, it quickly increases speed to the synchronous speed and locks onto the rotating stator field. Note that the damper windings have no induced currents as the synchronous motor rotates at the synchronous speed. The damper windings have another function as they help keep the synchronous motor in synchronism. If the synchronous motor speed increases or decreases away from the synchronous speed, currents are induced in the damper windings that tend to oppose the change in speed.