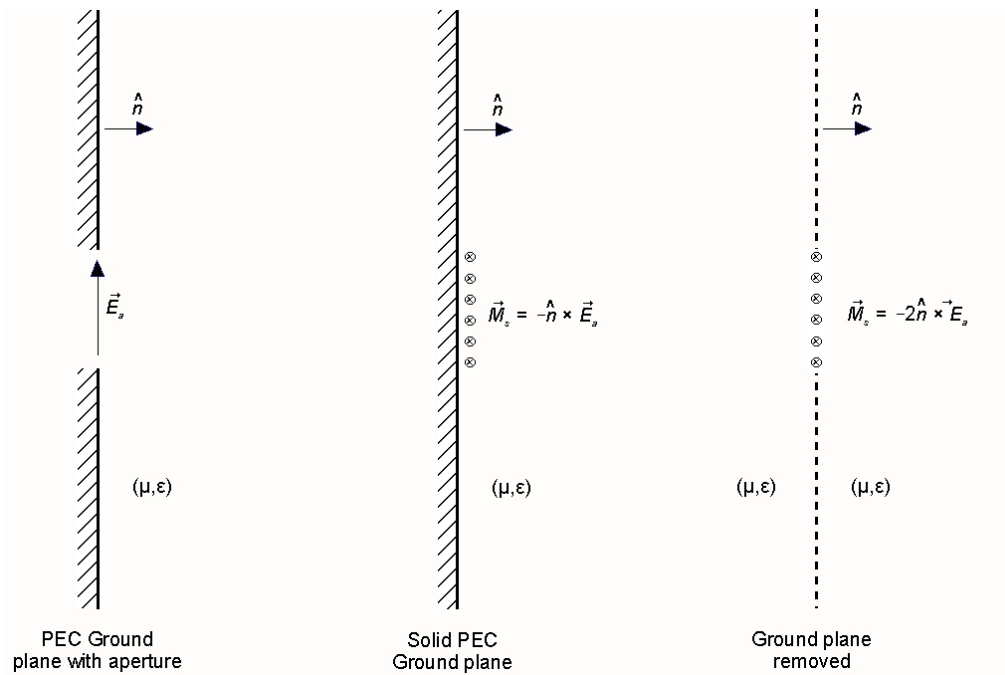


## Aperture Antennas

An aperture antenna contains some sort of opening through which electromagnetic waves are transmitted or received. Examples of aperture antennas include slots, waveguides, horns, reflectors and lenses. Aperture antennas are commonly used in aircraft or spacecraft applications. The aperture can be mounted flush with the surface of the vehicle, and the opening can be covered with a dielectric which allows electromagnetic energy to pass through. The fields radiated by aperture antennas are typically determined using a special case of the *field equivalence principle*.

Consider an aperture (opening) in an otherwise closed PEC, through which EM radiation occurs. According to the field equivalence principle, the fields external to the PEC can be determined using an equivalent geometry. The equivalent geometry consists of a magnetic surface current (related to the aperture field) located on the surface of the closed PEC (the aperture is covered by a PEC surface). For an aperture in a ground plane, image theory can then be applied, resulting in a magnetic surface current radiating in a homogeneous medium, which greatly simplifies the analysis.



## Open Ended Rectangular Waveguide

Consider an open-ended rectangular waveguide which connects to a conducting ground plane which covers the  $x$ - $y$  plane. If we assume that the waveguide carries only the dominant  $TE_{10}$  mode, the field distribution in the aperture of the waveguide is

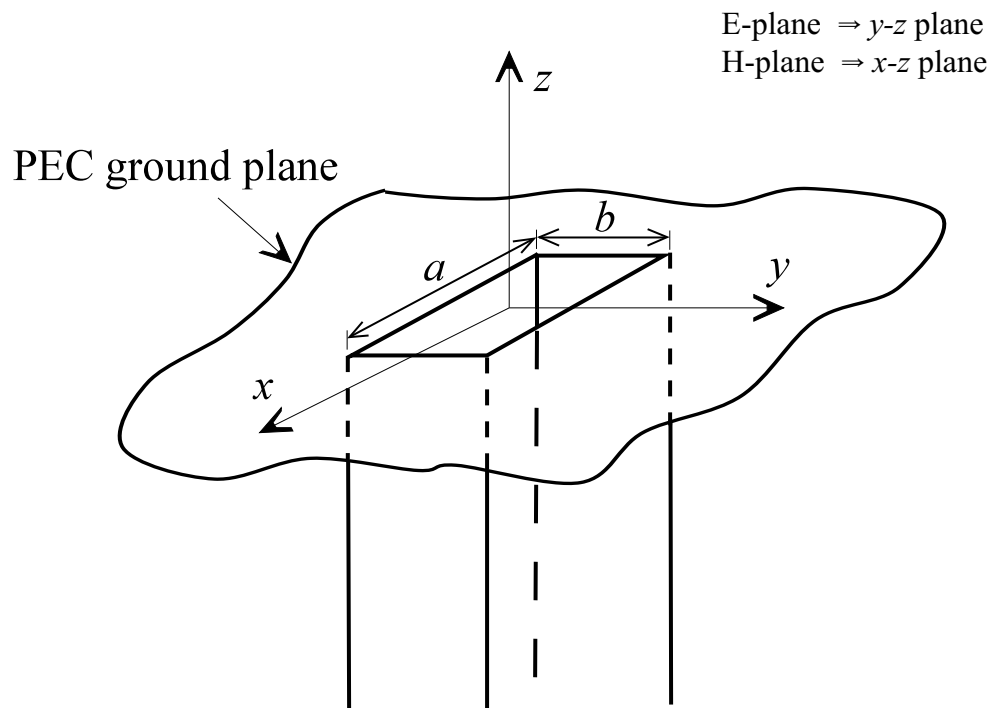
$$\mathbf{E}_a = E_o \cos\left(\frac{\pi x}{a}\right) \mathbf{a}_y \quad \left\{ -\frac{a}{2} \leq x \leq \frac{a}{2}, -\frac{b}{2} \leq y \leq \frac{b}{2} \right\}$$

$$\mathbf{H}_a = \frac{-\gamma_{10}}{j\omega\mu} E_o \cos\left(\frac{\pi x}{a}\right) \mathbf{a}_x$$

where

$$\gamma_{10} = jk \sqrt{1 - \left(\frac{f_{c10}}{f}\right)^2}$$

$$f_{c10} = \frac{1}{2a\sqrt{\mu\epsilon}}$$



According to the field equivalence principle, the equivalent magnetic surface current radiating in free space is given by

$$\mathbf{M}_s = -2\mathbf{a}_z \times \mathbf{E}_a = 2E_o \cos\left(\frac{\pi x}{a}\right) \mathbf{a}_x \quad \left\{ -\frac{a}{2} \leq x \leq \frac{a}{2}, -\frac{b}{2} \leq y \leq \frac{b}{2} \right\}$$

The resulting radiated far fields are

$$E_r = H_r = 0$$

$$E_\theta = -j \frac{abkE_o e^{-jkr}}{4r} \sin\varphi \frac{\cos\left(\frac{ka}{2} \sin\theta \cos\varphi\right) \sin\left(\frac{kb}{2} \sin\theta \sin\varphi\right)}{\left(\frac{ka}{2} \sin\theta \cos\varphi\right)^2 - \left(\frac{\pi}{2}\right)^2} \frac{kb}{2} \sin\theta \sin\varphi$$

$$E_\varphi = -j \frac{abkE_o e^{-jkr}}{4r} \cos\theta \cos\varphi \frac{\cos\left(\frac{ka}{2} \sin\theta \cos\varphi\right) \sin\left(\frac{kb}{2} \sin\theta \sin\varphi\right)}{\left(\frac{ka}{2} \sin\theta \cos\varphi\right)^2 - \left(\frac{\pi}{2}\right)^2} \frac{kb}{2} \sin\theta \sin\varphi$$

$$H_\theta = -\frac{E_\varphi}{\eta}$$

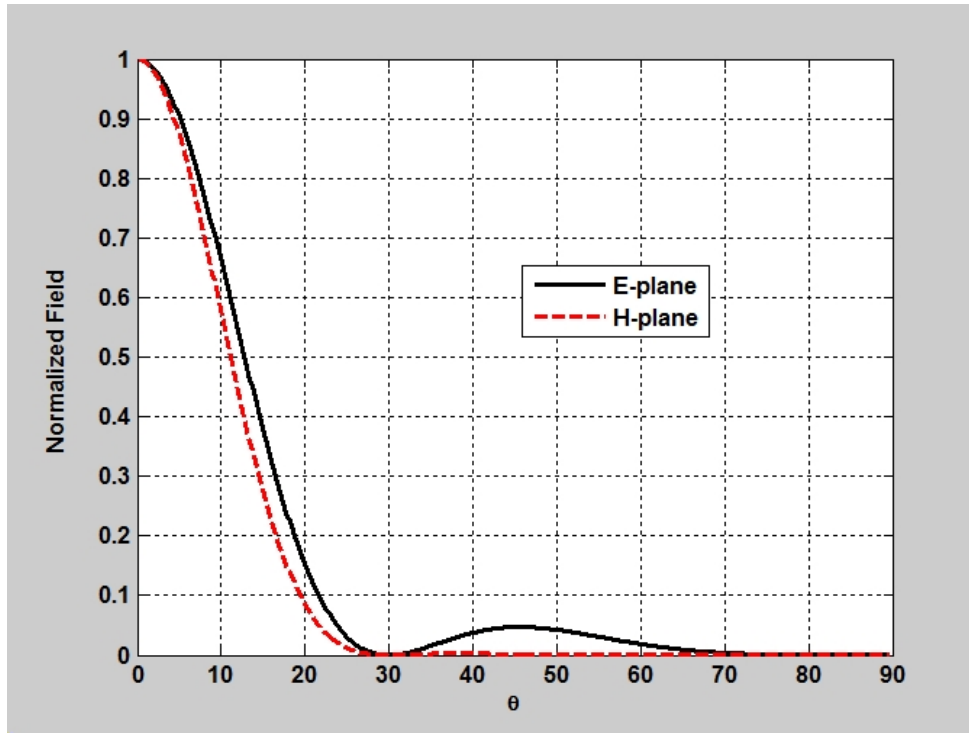
$$H_\varphi = \frac{E_\theta}{\eta}$$

The fields in the  $E$ -plane ( $\varphi = 90^\circ$ ) and  $H$ -plane ( $\varphi = 0^\circ$ ) reduce to

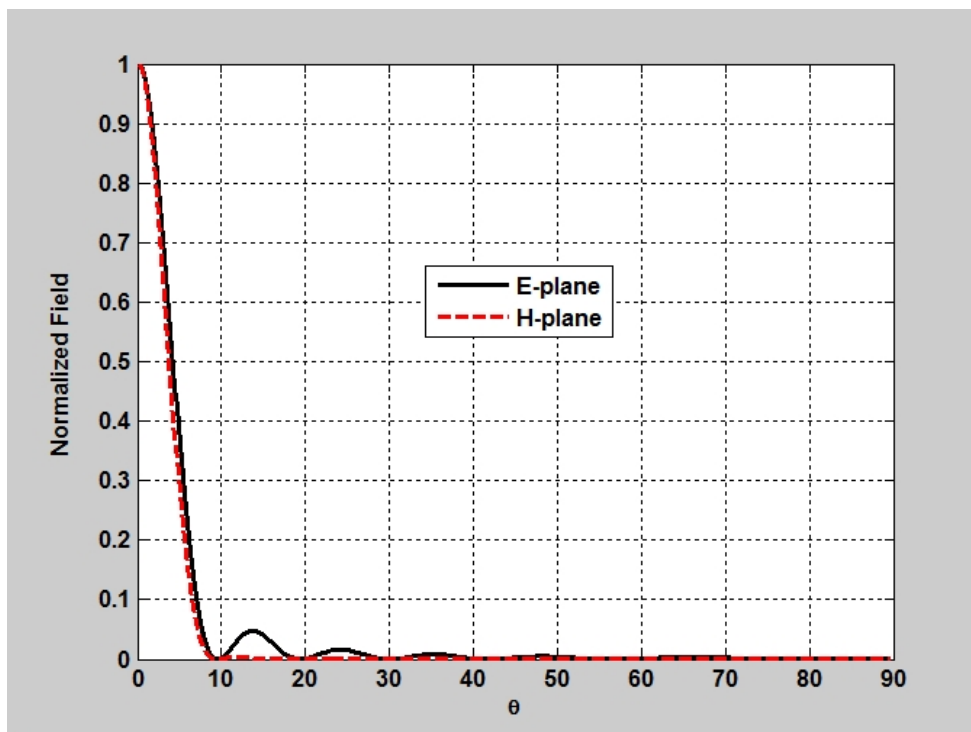
$$E_\theta = j \frac{abkE_o e^{-jkr}}{4r} \frac{4}{\pi^2} \frac{\sin\left(\frac{kb}{2} \sin\theta\right)}{\frac{kb}{2} \sin\theta} \quad (\varphi = 90^\circ)$$

$$E_\varphi = -j \frac{abkE_o e^{-jkr}}{4r} \cos\theta \frac{\cos\left(\frac{ka}{2} \sin\theta\right)}{\left(\frac{ka}{2} \sin\theta\right)^2 - \left(\frac{\pi}{2}\right)^2} \quad (\varphi = 0^\circ)$$

$$a = 3\lambda, b = 2\lambda$$



$$a = 9\lambda, b = 6\lambda$$

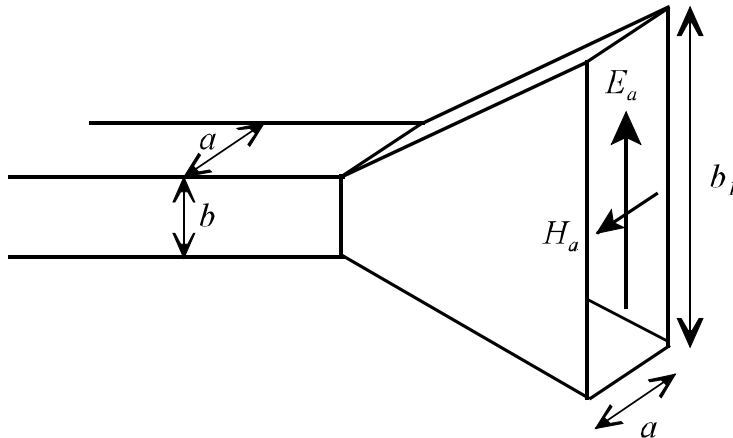


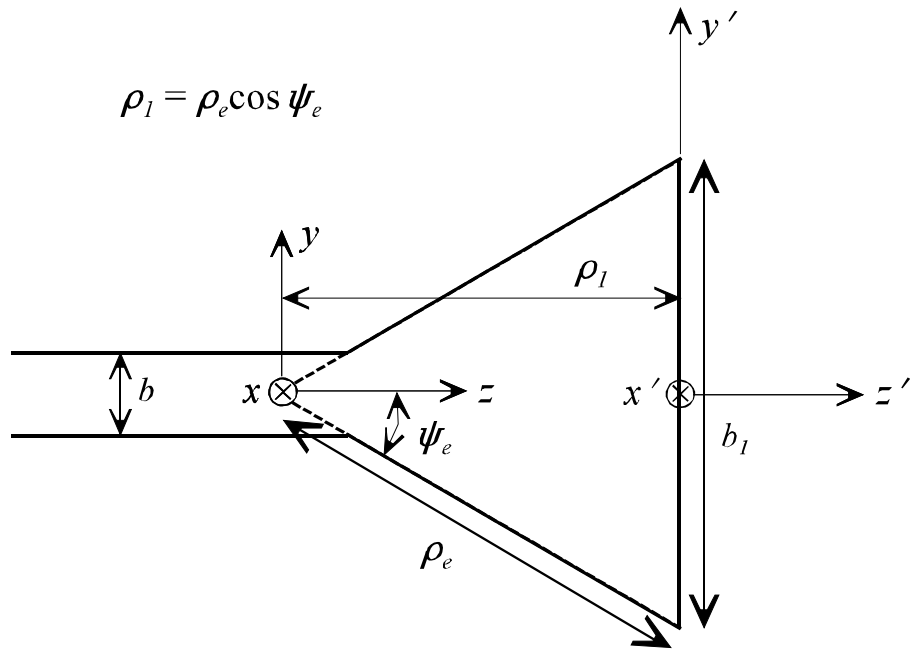
## Horn Antennas

The horn antenna represents a transition or matching section from the guided mode inside the waveguide to the unguided (free-space) mode outside the waveguide. The horn antenna, as a matching section, reduces reflections and leads to a lower standing wave ratio. There are three basic types of horn antennas: (a.) the  $E$ -plane sectoral horn (flared in the direction of the  $E$ -plane only), (b.) the  $H$ -plane sectoral horn (flared in the direction of the  $H$ -plane only), and (c.) the pyramidal horn antenna (flared in both the  $E$ -plane and  $H$ -plane). The flare of the horns considered here is assumed to be linear although some horn antennas are formed by other flare types such as an exponential flare.

The horn antenna is mounted on a waveguide that is almost always excited in single-mode operation. That is, the waveguide is operated at a frequency above the cutoff frequency of the  $TE_{10}$  mode but below the cutoff frequency of the next highest mode.

### $E$ -Plane Sectoral Horn





E-plane Sectoral Horn E-plane Far Field ( $\phi = \pi/2$ )

$$E_r = E_\phi = 0$$

$$E_\theta = j \frac{\alpha \sqrt{\pi k \rho_1} E_1 e^{-jkr}}{8r} \left\{ \left( \frac{2}{\pi} \right)^2 e^{j(k\rho_1 \sin^2 \frac{\theta}{2})} (1 + \cos\theta) F(t_1', t_2') \right\}$$

$$F(t_1, t_2) = [C(t_2) - C(t_1)] - j[S(t_2) - S(t_1)]$$

$$C(t) = \int_0^t \cos\left(\frac{\pi x^2}{2}\right) dx \quad (\text{cosine Fresnel integral})$$

$$S(t) = \int_0^t \sin\left(\frac{\pi x^2}{2}\right) dx \quad (\text{sine Fresnel integral})$$

$$t_1' = \sqrt{\frac{k}{\pi \rho_1}} \left( -\frac{b_1}{2} - \rho_1 \sin\theta \right) \quad t_2' = \sqrt{\frac{k}{\pi \rho_1}} \left( \frac{b_1}{2} - \rho_1 \sin\theta \right)$$

*E*-plane Sectoral Horn *H*-plane Far Field ( $\varphi = 0$ )

$$E_r = E_\theta = 0$$

$$E_\varphi = -j \frac{a \sqrt{\pi k \rho_1} E_1 e^{-jkr}}{8r} \left\{ (1 + \cos\theta) \left[ \frac{\cos\left(\frac{ka}{2} \sin\theta\right)}{\left(\frac{ka}{2} \sin\theta\right)^2 + \left(\frac{\pi}{2}\right)^2} F(t_1'', t_2'') \right] \right\}$$

$$t_1'' = \sqrt{\frac{k}{\pi \rho_1}} \left( -\frac{b_1}{2} \right)$$

$$t_2'' = \sqrt{\frac{k}{\pi \rho_1}} \left( \frac{b_1}{2} \right)$$

The directivity of the *E*-plane sectoral horn ( $D_E$ ) is given by

$$D_E = \frac{64 a \rho_1}{\pi \lambda b_1} \left[ C^2 \left( \frac{b_1}{\sqrt{2\lambda \rho_1}} \right) + S^2 \left( \frac{b_1}{\sqrt{2\lambda \rho_1}} \right) \right]$$

A plot of the *E*-plane and *H*-plane patterns for the *E*-plane horn shows that the *H*-plane pattern is much broader than the *E*-plane pattern. Thus, the *E*-plane sectoral horn tends to focus the beam of the antenna in the *E*-plane (see Figures 13.3 and 13.4). Design curves for the *E*-plane sectoral horn are given in Figure 13.8.

Example ( $E$ -plane sectoral horn design, Problem 13.6)

An  $E$ -plane horn is fed by a WR 90 (X-band) rectangular waveguide ( $a = 2.286$  cm,  $b = 1.016$  cm). Design the horn so that its maximum directivity at 11 GHz is 30 (14.77 dB).

$$\lambda = \frac{3 \times 10^8}{11 \times 10^9} = 0.02727 \text{ m} = 2.727 \text{ cm}$$

$$\frac{\lambda}{a} = \frac{2.727}{2.286} = 1.19 \quad \frac{\lambda}{a} D_E = 1.19 \times 30 = 35.79$$

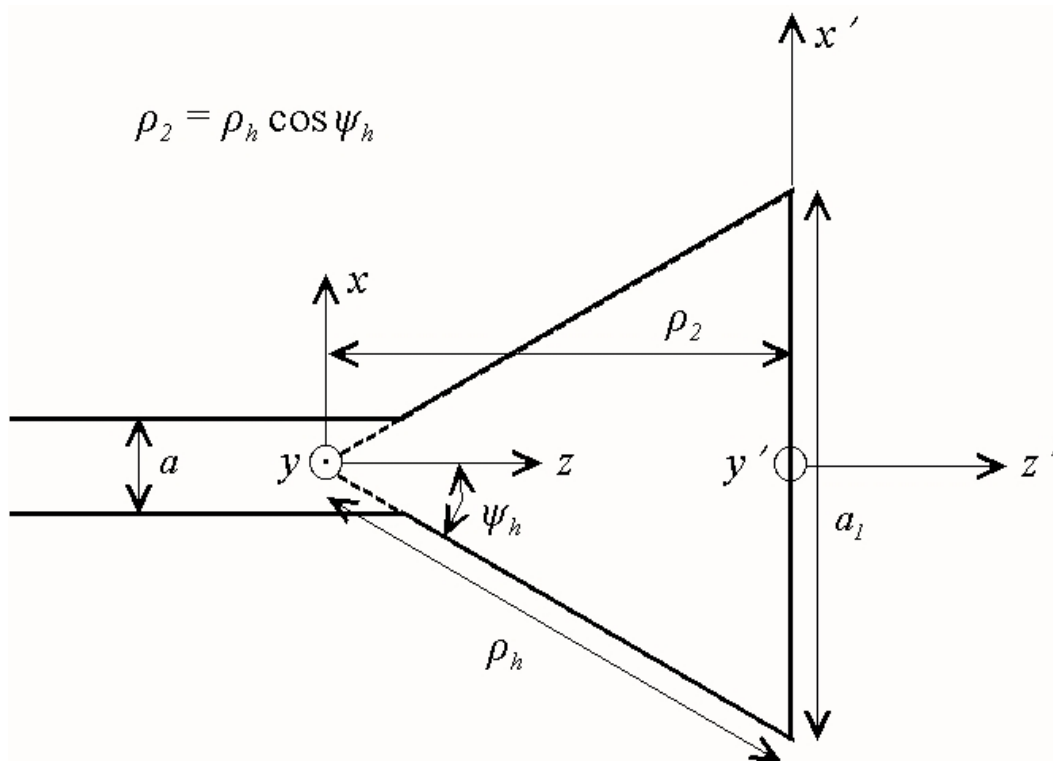
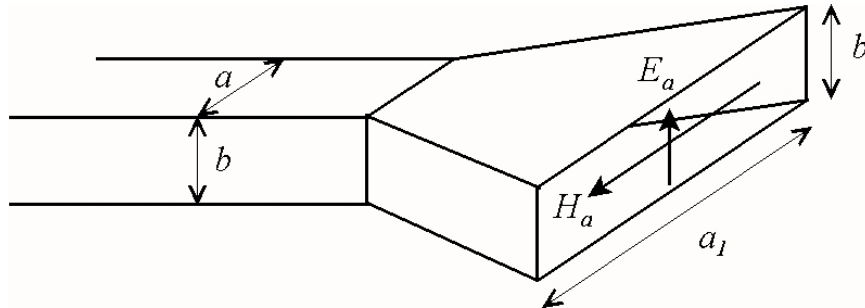
From Figure 13.8  $\Rightarrow \rho_1 = 10\lambda$  yields  $\frac{\lambda}{a} D_E \approx 36$  at  $b_1 \approx 4.5\lambda$

$$\psi_e = \tan^{-1} \left( \frac{b_1/2}{\rho_1} \right) = \tan^{-1}(0.225) = 12.68^\circ$$

$$\psi_e = 25.36^\circ$$



## H-Plane Sectoral Horn



H-plane Sectoral Horn E-plane Far Field ( $\varphi = \pi/2$ )

$$E_r = E_\varphi = 0$$

$$E_\theta = j \frac{b \sqrt{k \rho_2} E_2 e^{-jkr}}{8r \sqrt{\pi}} \times \left\{ (1 + \cos\theta) \frac{\sin Y}{Y} \left[ e^{jf_1} F(t_1', t_2') + e^{jf_2} F(t_1'', t_2'') \right] \right\}$$

$$Y = \frac{kb}{2} \sin\theta \quad f_1 = \frac{k_x'^2 \rho_2}{2k} \quad f_2 = \frac{k_x''^2 \rho_2}{2k}$$

$$k_x' = \frac{\pi}{a_1} \quad k_x'' = -\frac{\pi}{a_1}$$

H-plane Sectoral Horn H-plane Far Field ( $\varphi = 0$ )

$$E_r = E_\theta = 0$$

$$E_\varphi = j \frac{b \sqrt{k \rho_2} E_2 e^{-jkr}}{8r \sqrt{\pi}} \left\{ (1 + \cos\theta) \left[ e^{jf_1} F(t_1', t_2') + e^{jf_2} F(t_1'', t_2'') \right] \right\}$$

$$k_x' = k \sin\theta + \frac{\pi}{a_1} \quad k_x'' = k \sin\theta - \frac{\pi}{a_1}$$

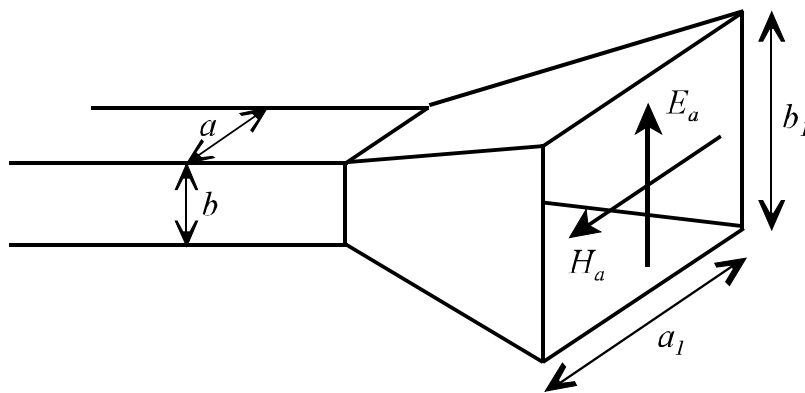
The directivity of the H-plane sectoral horn ( $D_H$ ) is given by

$$D_H = \frac{4\pi b \rho_2}{\lambda a_1} \left\{ [C(u) - C(v)]^2 + [S(u) - S(v)]^2 \right\}$$

$$u = \frac{1}{\sqrt{2}} \left( \frac{\sqrt{\lambda \rho_2}}{a_1} + \frac{a_1}{\sqrt{\lambda \rho_2}} \right) \quad v = \frac{1}{\sqrt{2}} \left( \frac{\sqrt{\lambda \rho_2}}{a_1} - \frac{a_1}{\sqrt{\lambda \rho_2}} \right)$$

A plot of the  $E$ -plane and  $H$ -plane patterns for the  $H$ -plane horn shows that the  $E$ -plane pattern is much broader than the  $H$ -plane pattern. Thus, the  $H$ -plane sectoral horn tends to focus the beam of the antenna in the  $H$ -plane (see Figures 13.11 and 13.12). Design curves for the  $H$ -plane sectoral horn are given in Figure 13.16.

### Pyramidal Horn



Based on the pattern characteristics of the  $E$ -plane and  $H$ -plane sectoral horns, the pyramidal horn should focus the beam patterns in both the  $E$ -plane and the  $H$ -plane. In fact, the  $E$ -plane and  $H$ -plane patterns of the pyramidal horn are identical to the  $E$ -plane pattern of the  $E$ -plane sectoral horn and the  $H$ -plane pattern of the  $H$ -plane sectoral horn, respectively (See Figure 13.19).

The directivity of the pyramidal horn ( $D_P$ ) can be written in terms of the directivities of the  $E$ -plane and  $H$ -plane sectoral horns:

$$D_P = \frac{\pi \lambda^2}{32 a b} D_E D_H$$