Spectral Variation Alleviation by Low-Rank Matrix Approximation for Hyperspectral Image Analysis

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Abstract—Spectral variation is profound in remotely sensed images due to variable imaging conditions. The wide presence of such spectral variation degrades the performance of hyperspectral analysis, such as classification and spectral unmixing. In this letter, $\ell_1$-based low-rank matrix approximation is proposed to alleviate spectral variation for hyperspectral image analysis. Specifically, hyperspectral image data are decomposed into a low-rank matrix and a sparse matrix, and it is assumed that intrinsic spectral features are represented by the low-rank matrix and spectral variation is accommodated by the sparse matrix. As a result, the performance of image data analysis can be improved by working on the low-rank matrix. Experiments on benchmark hyperspectral data sets demonstrate the performance of classification, and spectral unmixing can be clearly improved by the proposed approach.

Index Terms—Classification, hyperspectral imagery, low-rank matrix approximation, spectral unmixing, spectral variation.

I. INTRODUCTION

HYPERSPECTRAL remote sensing technology has been demonstrated to be a powerful tool to discriminate different materials since collected hyperspectral images have very high spectral resolution. However, due to the fact that the solar illumination lacks uniformity and consistency over large areas, even homogeneous ground objects often present different spectral signatures, which can be considered as within-class spectral variation. Within-class spectral variation may also result in between-class similarity, which means that different materials may present similar spectral signatures, thus degrading the performance of hyperspectral image analysis.

In previous studies, three types of approaches have been utilized to accommodate within-class spectral variation: 1) using spatial context; 2) modeling the spectrum of a pure ground material by a probability density function (pdf); and 3) constructing a spectral library with many spectra per class (or endmember bundles). Spatial context has been widely utilized to improve the performance of hyperspectral image analysis by alleviating within-class spectral variation. For example, the support vector machine (SVM) with composite kernels (SVM-CK) classifier incorporates spatial information directly in the SVM kernels [1] for classification, and average signatures from spatial neighbors can be used to improve classification accuracy [2]. In spectral unmixing, the automated morphological endmember extraction (EE) algorithm extended morphological transformations by exploring spatial context based on the generalized mathematical morphology theory [3], the spatial purity-based EE algorithm measured the purity of pixels according to their spatial context [4], and spatial–spectral information was jointly considered in EE in [5]. However, spatial context can alleviate spectral variation only in limited areas of an image. In order to accommodate the spectral variation statistically, the spectral response of a ground material is characterized by a pdf. The expectation maximization algorithm [6] and Bayesian self-organizing map [7] can be utilized to estimate parameters in endmember distribution and their corresponding fractional abundance. However, it is difficult to know the true pdf of endmembers in an image, although it is often assumed to be Gaussian. In addition, the inaccurate estimation of parameters in pdf also affects the subsequent analysis. In a spectral library, each pure ground material is represented by a batch of spectra to model within-class spectral variation, and sparse related theory has been adopted for hyperspectral image analysis. For example, in hyperspectral image classification, sparse representation and collaborative representation methods considered training samples as a spectral library to accommodate within-class spectral variation [8], [9]; in sparse unmixing algorithms, both an available spectral library (such as the U.S. Geological Survey (USGS) library [10]) and a field spectral library have been adopted to accommodate within-class spectral variation [11], [12], in which the field spectral library offered superior accuracy. Endmember bundle extraction can be also found in [13] and [14]. However, the performance may be limited if the spectral library or endmember bundles cannot well represent spectral variation in different areas of an image.

Recently, low-rank modeling has achieved great success in various fields [15], [16]. Specifically, in hyperspectral image processing, Zhao and Yang adopted sparse representation with a low-rank constraint for hyperspectral image denoising [17]. The noise was reduced by sparsely representing the image with respect to a learned dictionary. In addition, a low-rank constraint was imposed to remove global correlation and redundancy of the image. Lu et al. proposed a graph-regularized low-rank representation for destriping of hyperspectral images [18]. The denoising algorithms focus on a certain kind of noise in an image, which is actually a source of spectral variation. The intrinsic representations of a hyperspectral image are also proposed recently [19], [20]. However, a few of features, instead
of a cleaner image with the same size of the original one, are resulted in. A low-rank subspace representation algorithm was also proposed to estimate the number of signal subspaces in hyperspectral images. This algorithm assumed that hyperspectral data are drawn as a union from multiple subspaces [21]. In this letter, a simple but effective spectral variation alleviation algorithm is proposed by performing a low-rank approximation of a hyperspectral image. In particular, the original image data is decomposed into a low-rank matrix and a sparse matrix, and it is assumed that intrinsic spectral features are represented by the low-rank matrix and spectral variation is accommodated by the sparse matrix. As a result, the influence of spectral variation can be alleviated by performing low-rank approximation as a preprocessing step. Experimental results on both classification and spectral unmixing demonstrate the effectiveness of the proposed approach.

II. LOW-RANK MATRIX APPROXIMATION OF HYPERSPECTRAL IMAGES

A spectral signature is known as a unique “fingerprint” that can be used to discriminate different materials. However, even for the same material, its spectral signature is varied due to variable illumination, environmental, and atmospheric conditions. Consequently, an acquired spectral pixel often consists of two parts: intrinsic spectral signature and spectral variation, which can be formulated as

$$r = s + v$$

where \(r, s, v \in \mathbb{R}^{b \times 1}\) represent the observed spectral response, intrinsic spectral signature, and spectral variation within the considered pixel, respectively, and \(b\) is the number of bands. If spectral variation is ignored when analyzing the pixel, errors are inevitably introduced and propagated throughout hyperspectral image analysis. To alleviate such spectral variation, all the pixels in an image can be considered together. Thus, the observed hyperspectral image can be represented as

$$R = S + V$$

where \(R \in \mathbb{R}^{b \times n}\) is the entire data matrix, \(S, V \in \mathbb{R}^{b \times n}\) represent the intrinsic spectral signature matrix and spectral variation matrix, respectively, and \(n\) represents the number of pixels in the image. As a result, if the analysis is carried out on the intrinsic spectral signature matrix \(S\) instead of the observed pixel matrix \(R\), the influence of spectral variation can be alleviated.

In the problem of classification, all the pixels in an image are classified into a small number of classes. Moreover, the number of classes is much smaller than the observed dimensionality of the data (equal to the number of observed bands). In spectral unmixing, all the mixtures are modeled as a linear/nonlinear combination of a small number of endmembers. Similarly, the number of endmembers is also much smaller than the number of bands. Fig. 1 quantitatively illustrates the low-rank characteristics of hyperspectral images, where the normalized singular values (with respect to the largest one) of two popular hyperspectral images are given. It is observed that the singular values decrease very rapidly, and the sixth principal component is only 1% of the first one, indicating the low-rank property. As a result, the intrinsic spectral signature matrix of a hyperspectral image presents a low-rank property. Thus, solving the intrinsic spectral signature matrix \(S\) is a low-rank matrix approximation problem

$$\begin{align*}
\min_{S, V} & \quad \text{rank}(S) \\
\text{s.t.} & \quad S + V = R
\end{align*}$$

where \(\text{rank}(\cdot)\) computes the rank of a matrix. In order to well formulate the spectral variation alleviation problem, an extra constraint for the spectral variation matrix \(V\) is imposed. According to (3), the spectral variation matrix \(V = R - S\) can be viewed as a data acquisition error. Under the assumption of Gaussian noise, the \(\ell_2\) norm, or more exactly the Frobenius norm in matrix form, is often adopted since the resulting optimization problem is convex and easy to solve [22]. However, for hyperspectral data, the variation is caused by not only the sensor noise but also the variable conditions during data acquisition that may not follow a Gaussian distribution. As a result, a more robust measurement criterion, namely, the \(\ell_1\) norm, is adopted to model the spectral variation part. Thus, the following optimization problem is constructed to obtain the spectral signature matrix \(S\):

$$\begin{align*}
\min_{S, V} & \quad \text{rank}(S) + \lambda \cdot \|V\|_1 \\
\text{s.t.} & \quad S + V = R
\end{align*}$$

where \(\|\cdot\|_1\) represents the \(\ell_1\) norm of a matrix that calculates the summation of the absolute values of all the elements in the matrix and \(\lambda\) is a regularization parameter. By adopting the \(\ell_1\) norm, the proposed model is also capable of handling outliers [22], [23]; thus, the performance can be further improved for hyperspectral data with outliers, resulting in more robust analysis results.

The optimization problem in (4) is difficult to solve due to the discrete and nonconvex nature of the rank function. Fortunately, as suggested in [24], the following convex optimization provides an appropriate surrogate for (4):

$$\begin{align*}
\min_{S, V} & \quad \|S\|_* + \lambda \cdot \|V\|_1 \\
\text{s.t.} & \quad S + V = R
\end{align*}$$

where \(\|\cdot\|_*\) represents the nuclear norm of a matrix, i.e., the summarization of singular values. The convex optimization problem defined by (5) is known as the robust principal component analysis (RPCA) in previous research [15]. The inexact-augmented-Lagrange-multiplier algorithm [15] is adopted to solve the convex optimization problem defined by (5). In the proposed approach, \(\lambda\) is selected to be the one producing the overall satisfactory performance, which is around \(1/\sqrt{n}\).
according to the work in [15]. After the low-rank matrix approximation of hyperspectral data is obtained, the influence of spectral variation can be alleviated by conducting analysis on the low-rank matrix $S$, instead of the original data $R$.

The geometric interpretation of the proposed approach is illustrated ideally in Fig. 2. Fig. 2(a) shows a two-cluster case in a 2-D space. By the use of low-rank recovery for spectral variation alleviation, all the revised pixels in the same class may become closer to each other, while those in different classes move far away from each other. Thus, the performance of classification will be improved. For linear unmixing, Fig. 2(b) illustrates a three-endmember case. According to the linear mixture assumption, all the mixed pixels fall into a simplex defined by endmembers. However, due to the presence of spectral variation, many marginal pixels may fall outside the simplex. Under the worst case, false “endmembers” may be extracted to form a simplex covering all the pixels. By alleviating spectral variation through low-rank matrix approximation, all the revised pixels may clearly fall inside the true simplex.

III. EXPERIMENTS

A. Experimental Results of Classification

In this experiment, the hyperspectral data set acquired over the Indian Pines area in northwestern Indiana by the National Aeronautics and Space Administration (NASA) Airborne Visible/Infrared Imaging Spectrometer (AVIRIS) sensor in June 1992 is used. This data set contains $145 \times 145$ pixels with a ground resolution of $17$ m. Of the $224$ atmospherically corrected channels, $200$ bands are used after removing the channels associated with $H_2O$ and OH absorption features near $1400$ and $1900$ nm. This image scene contains $16$ different land-cover classes of agriculture. The average of the standard variation among all the pixels in the same class, which measures the deviation from the average spectrum of a class, is adopted to quantitatively measure spectral variation. As shown in Fig. 3, it can be observed that spectral variation has been significantly decreased by the proposed approach.

In order to further evaluate the performance improvement, the classification results on the original data, the low-rank recovered data by the proposed algorithm that imposes an $\ell_1$-based sparse constraint on the spectral variation matrix, and the low-rank recovered data by imposing an $\ell_2$ constraint on the spectral variation matrix are evaluated. Under the assumption of Gaussian noise, the truncated singular value decomposition is adopted to approximate hyperspectral data with an $\ell_2$ constraint on the spectral variation matrix. In this experiment, $95\%$ of the largest singular values are retained. The popular K-nearest neighbor (KNN) algorithm with $k = 1$ and SVM and SVM-CK [1] algorithms are adopted as classifiers. The number of training samples in different classes is shown in Table I. The overall classification accuracy and Kappa coefficient are adopted as quantitative metrics to evaluate the performance of classification. Table I lists the average results of $50$ trials by randomly selected training samples from labeled samples. Obviously, the performance of classification has been improved by the proposed $\ell_1$-based low-rank recovery. Moreover, the results of the proposed $\ell_1$-based low-rank recovery are even better than that by the $\ell_2$-based low-rank recovery, indicating that the $\ell_1$ norm can better model spectral variation than the $\ell_2$ norm. The results of the $\ell_2$-based low-rank recovery are even worse than that of the original data, indicating that the $\ell_2$ constraint is not suitable to model the spectral variation matrix. In the proposed algorithm, pixels in the same class are more likely to present near each other since the within-class spectral variation has been alleviated; therefore, even for KNN with one pixel being selected as neighbor from the training samples, classification accuracy can be greatly improved. In addition, since spatial information is utilized in the SVM-CK, classification performance can also be further improved.

The performance of classification on another popular hyperspectral data set, namely, the Pavia University data set collected by the Reflective Optics System Imaging Spectrometer sensor, is also evaluated. The data have a spectral coverage from $0.43$ to $0.86$ $\mu$m and a spatial resolution of $1.3$ m. A subset containing $610 \times 340$ pixels over $103$ bands is selected. In the proposed approach, the image is divided into $155 \times 170$ blocks to perform low-rank recovery. Table II lists the average results of $50$ trials by randomly selected $50$ training samples per class. It is also confirmed that the performance of classification can be improved by the proposed $\ell_1$-based low-rank-based data recovery and the $\ell_1$-based constraint can better model spectral variation than the $\ell_2$-based one.

B. Experimental Results of Spectral Unmixing

The well-known AVIRIS data set over the Cuprite areas in Nevada, USA,$^1$ which has $224$ channels ranging from $370$ to $2510$ nm with a ground instantaneous field of view of $20$ m, is used in this experiment. The cropped image corresponds to a $350 \times 350$ pixel subset of the sector labeled

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as f97061901p02_r02_sc03.a.rfl in the online data. In this experiment, only 188 bands are adopted after removing bands 1–2, 105–115, 150–170, and 223–224 due to water absorption and low signal-to-noise ratio in those bands. The Cuprite site has been well understood mineralogically with reliable ground truth [10].

In order to further evaluate the performance of the proposed algorithm, we test the performance of spectral unmixing on the original data, its revised version by the proposed $\ell_1$-based low-rank matrix approximation, and that by the $\ell_2$-based low-rank matrix approximation, respectively. Both the performances of EE and abundance estimation (AE) are considered. Two popular EE algorithms are adopted to extract endmembers: vertex component analysis (VCA) [25] algorithm and N-FINDR algorithm [26]. Totally, 16 endmembers are extracted from this area according to one of the popular virtual dimensionality estimation algorithm, namely, the Hysime algorithm [27]. In this field 13 highly representative minerals are considered: Alunite, Andradite, Buddingtonite, Calcite, Chaledony, Dumortierite, Hematite, Jarosite, Kaolinite, Montmorillonite, Muscovite, Sphene, and Pyrophyllite. Their corresponding laboratory spectra in the USGS spectral library [10], which are convolved in accordance with AVIRIS wavelength specifications, are utilized as ground-truth spectra. In order to perform quantitative evaluation, the spectral angle distance (SAD) values (in degrees), which are obtained by comparing the USGS library spectra [10] with their corresponding endmembers extracted, are adopted for quantitative assessment. The results of EE on the original Cuprite data set and its improved versions by the low-rank matrix approximation are listed in Table III. The performance of EE on the revised data with the proposed $\ell_1$-based low-rank matrix approximation slightly outperforms that on the other two kinds of data, indicating that the performance of EE can be improved by alleviating spectral variation with the proposed $\ell_1$-based low-rank recovery. In order to further evaluate the performance of unmixing, the fully constrained least squares (FCLS) algorithm [28] is then adopted to estimate the abundance maps for these estimated endmembers. The reconstruction error of pixels is adopted to further quantitatively evaluate the unmixing performance. The average of per-pixel Root Mean Squared Error (RMSE) is listed in Table III. The results of AE are consistent with that of EE, indicating that the proposed $\ell_1$-based low-rank recovery is effective to alleviate spectral variation for spectral unmixing. In particular, the per-pixel RMSE has been sharply decreased by the proposed $\ell_1$-based low-rank recovery when N-FINDR and FCLS algorithms are adopted for spectral unmixing.
In this letter, spectral variation in hyperspectral images is alleviated by $\ell_1$-based low-rank recovery. In particular, low-rank-matrix-approximation-based RPCA is proposed as a preprocessing step to alleviate spectral variation. Experimental results on both classification and spectral unmixing have demonstrated that the performance of hyperspectral image analysis can be improved after alleviating spectral variation using the proposed approach.

### IV. CONCLUSION

In this letter, spectral variation in hyperspectral images is alleviated by $\ell_1$-based low-rank recovery. In particular, low-rank-matrix-approximation-based RPCA is proposed as a preprocessing step to alleviate spectral variation. Experimental results on both classification and spectral unmixing have demonstrated that the performance of hyperspectral image analysis can be improved after alleviating spectral variation using the proposed approach.

### REFERENCES


