

Shannon-Fano-Elias Coding

Example:

$$m = 5, \quad \mathcal{X} = \{1, 2, 3, 4, 5\}, \quad l_i = \lceil -\log p_i \rceil + 1, \quad \bar{F}(x_i) = \frac{F(x_{i-1}) + F(x_i)}{2}$$

x_i	p_i	$F(x_i)$	$\bar{F}(x_i)$	l_i	Code = $\lfloor \bar{F}(x_i) \rfloor_{l_i}$
1	0.2	0.2	$0.1 \approx 0.0001100_2$	4	0001
2	0.5	0.7	$0.45 \approx 0.0111001_2$	2	01
3	0.15	0.85	$0.775 \approx 0.1100011_2$	4	1100
4	0.10	0.95	$0.9 \approx 0.1110011_2$	5	11100
5	0.05	1.00	$0.975 \approx 0.1111100_2$	6	111110

Performance:

$$L(C) = 0.2 \cdot 4 + 0.5 \cdot 2 + 0.15 \cdot 4 + 0.10 \cdot 5 + 0.05 \cdot 6 = 3.2 \text{ bits}$$

Theoretical performance:

$$H(X) \approx 1.9232 \text{ bits}$$

$$H(X) \leq L(C) < H(X) + 2$$