1.1 \( (a) \) \[ x(t) = \rho_2(t) + \rho_4(t) \]

\( (b) \) \[ x(t) = \frac{4}{3} \left( 1 - \frac{1}{4}t \right) \rho_8(t) - \frac{1}{3} \left( 1 - \frac{1}{4}t \right) \rho_0(t) \]

\( (c) \) \[ x(t) = 2 \rho_{12}(t) + 2 \rho_6(t) + 2 \left( 1 - \frac{1}{3}t \right) \rho_6(t) \]

\( (d) \) \[ x(t) = 4 \rho_4(t) - 2 \left( 1 - \frac{1}{2}t \right) \rho_4(t) \]

1.4 \( (c) \)

1.17 \( (a) \) Causal and memoryless since the output \( y(t) \) at time \( t \) depends only on the input \( x\theta(t) \) at time \( t \).

\( (c) \) Same answer as part \( (a) \)\)

\( (e) \) Causal and has memory since the output \( y(t) \) at time \( t \) depends on the input \( x(\lambda) \) for \( \lambda = 0 \) to \( \lambda = t \).
1.19 (a) Let \( y(t) \) be the response to \( x(t) \).
So that \( y(t) = |x(t)| \). Then the response to \( -x(t) \) is \( y(t) = | -x(t) | = |x(t)| \), which is not equal to \( -y(t) \), and thus the system is not homogeneous, which shows that the system is not linear.

(c) Let \( y_1(t) \) be the response to \( x_1(t) \) and \( y_2(t) \) be the response to \( x_2(t) \). Then
\[
y_1(t) = (\sin t) x_1(t) \quad \text{and} \quad y_2(t) = (\sin t) x_2(t)
\]

Let \( \tilde{y}(t) \) denote the response to \( a x_1(t) + b x_2(t) \), where \( a \) and \( b \) are scalars. Then
\[
\tilde{y}(t) = (\sin t) [a x_1(t) + b x_2(t)]
= a (\sin t) x_1(t) + b (\sin t) x_2(t)
\]

Hence \( \tilde{y}(t) = a y_1(t) + b y_2(t) \), which shows that the system is linear.

(e) Let \( y_1(t) \) be the response to \( x_1(t) \) and \( y_2(t) \) be the response to \( x_2(t) \). Then
\[
y_1(t) = \int_0^t (t - \lambda) x_1(\lambda) \, d\lambda \quad \text{and} \quad y_2(t) = \int_0^t (t - \lambda) x_2(\lambda) \, d\lambda
\]
(continued on next page)
Let \( y(t) \) denote the response to \( ax(t) + bx_0(t) \) where \( a \) and \( b \) are scalars. Then

\[
y(t) = \int_0^t [(t-\lambda)(ax_0(\lambda) + bx_0(\lambda))] d\lambda
\]

By linearity of the integration operation,

\[
y(t) = \int_0^t [(t-\lambda)x_0(\lambda) d\lambda + b \int_0^t (t-\lambda)x_0(\lambda) d\lambda]
\]

Hence \( y(t) = ay(t) + by_0(t) \), which shows that the system is linear.

1.20 (a) Let \( y(t) \) be the response to \( x(t) \), so that \( y(t) = |x(t)| \). For any \( t_1 \), the response to \( x(t-t_1) \) is equal to \(|x(t-t_1)|\), which is equal to \( y(t-t_1) \), so the system is time invariant.

(c) For any \( t_1 \), the response to \( x(t-t_1) \) is equal to \((\sin t)(t-t_1)\), which is not equal to \( y(t-t_1) \), since \( y(t-t_1) \) is equal to \([\sin(t-t_1)]t-t_1\). Therefore, the system is time varying.
1.20 (e) Let \( \tilde{y}(t) \) denote the response to 
\( x_1(t-t_1) \). Then 
\[
\tilde{y}(t) = \int_{0}^{t} (t-\lambda) x(t-t_1) d\lambda
\]

However,
\[
\gamma(t-t_1) = \int_{0}^{t-t_1} (t-t_1-\lambda) x(\lambda) d\lambda
\]

Let \( \Delta = t_1 + \lambda \), so \( \Delta = t_1 \) when \( \lambda = 0 \),
and \( \Delta = t \) when \( \lambda = t-t_1 \).

Also \( \lambda = \Delta - t_1 \), and inserting this into
the integral expression for \( \gamma(t-t_1) \)
gives
\[
\gamma(t-t_1) = \int_{t-t_1}^{t} (t-\lambda) x(\lambda-t_1) d\lambda
\]

\[
= \int_{t-t_1}^{t} (t-\lambda) x(\lambda-t_1) d\lambda \neq \tilde{y}(t)
\]

Thus the system is time varying.
2.20 \( \frac{L di(t)}{dt} + R i(t) + \frac{1}{C} \int_0^t i(\lambda) d\lambda = x(t) \) (KCL)

(a) \( V_C(t) = \frac{1}{C} \int_{-\infty}^t i(\lambda) d\lambda \), so \( i(t) = C \frac{dV_C(t)}{dt} \)

\[ \Rightarrow L \frac{d^2V_C(t)}{dt^2} + R \frac{dV_C(t)}{dt} + \frac{1}{C} V_C(t) = \frac{1}{C} x(t) \]

(b) \( L \frac{d^2i(t)}{dt^2} + R \frac{di(t)}{dt} + \frac{1}{C} i(t) = \frac{d^2x(t)}{dt^2} \)

by differencing KCL equation

2.21 (see separate pdf)

2.24 (a) Summing currents we have

\[ \frac{1}{K} y(t) + \frac{1}{L} \int_{-\infty}^t y(\lambda) d\lambda = i(t) \]

Differentiating both sides yields

\[ \frac{1}{K} \frac{dy(t)}{dt} + \frac{1}{L} y(t) = \frac{di(t)}{dt} \]

So with \( R = L = 1 \), we have

\[ \frac{dy(t)}{dt} + y(t) = \frac{di(t)}{dt} \]